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A Computational Inverse Technique for Uncertainty Quantification in an Encounter Condition Identification Problem

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Abstract: A novel inverse technique is presented for quantifying the uncertainty of the identified the results in an encounter condition identification problem. In this technique, the polynomial response surface method based on the structure-selection technique is first adopted to construct the approximation model of the projectile/target system, so as to reduce the computational cost. The Markov Chain Monte Carlo method is then used to identify the encounter condition parameters and their confidence intervals based on this cheap approximation model with Bayesian perspective. The results are demonstrated through the simulated test cases, which show the utility and efficiency of the proposed technique. Since the uncertainty propagation in this identification process is efficiently explored, this technique can give us a clear indication of the degree to which we can trust estimates of the resulting encounter conditions.

Keywords: Inverse problems; Bayesian approach; Penetration; Encounter condition; Uncertainty quantification.

1 Introduction

The studies of the multilayer medium subjected to a projectile penetration are of significant interest in many engineering applications, e.g. protection engineering, aerospace engineering and civil engineering. In the projectile/multilayer medium system, striking velocity and oblique angle of the projectile are known as the encounter condition, while penetration depth and state angle are known as the projectile state. The encounter condition identification problem is usually defined as an inverse problem to determine the striking velocity and oblique angle of the projectile through the given projectile state data, in contrast with the forward problem

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which are concern with the estimation of the projectile state from the known encounter condition. The inverse problem in the projectile/target system is essential for understanding damage mechanisms of the multilayer medium, and discovering variety rule of the penetration event. Hence, it would be desirable to develop a reliable method to deal with this problem.

The inverse problem of determining the encounter condition which relates to the impact dynamics is really a challenging subject [Goldsmith (1999)]. One of the biggest difficulties is that the uncertainty widely exists in the projectile/target system. As in the real physical system, this penetration event displays random variations from many sources, including the geometry of the problem, material properties and boundary/initial conditions etc [Marin et al. (2006)]. This inherently nondeterministic nature of the problem calls for a characterization of the present uncertainty as a necessary step in determining the encounter condition. This is complicated by the fact that we should be answering the question: "how can we trust the resulting encounter condition parameters? And what is the confidence interval of this obtained solutions? " .

Therefore, we can easily found that the key features of the ideal method to determine the encounter condition would include (1) the method would be impervious to the uncertain system (i.e., error of measurement data, inaccuracy of forward model, etc.), (2) the approach would be give engineers a clear indication of the degree to which they can trust the obtained results, and (3) the ability to fast and accurately identify the encounter conditions from various projectile state data.

While there have been many types of methods proposed, there relatively few references related to the encounter condition identification problem in a projectile/target system. Traditionally, the experimental method [Forrestal et al. (2003)] provides the highest accuracy but has the disadvantage of being valid exclusively for the particular projectile/target system. The theoretical analysis [Li and Chen (2003)] formulates penetration problems by using forward equations based on some hypotheses to simplify the actual mechanisms of the penetration process. Nevertheless, this approach has the lower accuracy and each method has a specific application range. Numerical simulation can reproduce the detailed process of penetration by solving differential equations of mechanics. Unfortunately, the inaccurate modeling and the low computational efficiency for a complicated projectile/target model are the shortcoming of this approach [Gawin et al. (2011), Marin et al. (2006)]. More recently, a computational inverse technique is presented by [Zhang et al. (2012b)] to determine the encounter conditions from using the deterministic and uncertain projectile state data. While this technique met with some success, the study of the confidence estimates for the resulting parameters would be still needed.

In order to better assess the uncertainty in the estimates, we advocate a Bayesian approach to determine the encounter condition. In this approach, we estimate the entire posterior distribution of the encounter condition parameters. Therefore, we might choose the final estimate to be the mean of the posterior distribution or the maximum of the posterior distribution. The spread of the posterior additionally provides us with a measure of confidence in the estimates. In other word, it can provide a confidence interval for each encounter condition parameter. It is this flexibility that has largely motivated our use of Bayesian approach. Recently, Emerging studies attempt to apply the Bayesian approach in various engineering inverse problems. Yan et al. [Yan et al. (2011)] reconstruct the unknown heat source by using an augmented Tikhonov regularization method derived from a Bayesian perspective. Moore et al. [Moore et al. (2011)] and Nichols et al. [Nichols et al. (2011)] used a Bayesian approach to identify crack in a vibrating plate based on free-decay data, respectively. Zhang et al. [Zhang et al. (2012)] presented a Bayesian approach for force reconstruction which can deal with both measurement noise and model uncertainty. Work by [Zhang et al. (2012a)] presented a fast Bayesian approach based on the adaptive densifying approximation technique for parameter identification of computationally expensive models.

The purpose of this work is to present an efficient inverse technique based on Bayesian approach to rapidly identify the encounter condition parameters and accurately quantifying the uncertainty in the obtained results from the given noise added projectile state data. This technique provides an identification of the entire distribution of the encounter condition parameters. Hence, it is straight forward to estimate the means and the confidence intervals for each parameter. Nevertheless, a major limitation of Bayesian approach is its computationally expensive cost in treating this problem. Sampling method like standard Markov Chain Monte Carlo (MCMC) method for exploiting the posterior distribution commonly requires 10000-50000 samples for obtaining reliable results. Unfortunately, each single calculation of the FEM projectile/target model itself is real a daunting task. To overcome this difficulty, a polynomial response surface method based on the structure-selection technique is applied to obtain the approximation response of the projectile/target model. Thus, it is significantly faster than the actual FEM forward model. The obtained responses are then used in the Bayesian approach to fast determine the encounter condition parameters and assess their plausibility.

2 Statement of the problem

The present work only considers simulation results. The main aim is to see if the present inverse technique can successfully identify the mean of the encounter condition parameters and their confidence intervals from the known probability distribution of the projectile state data at the specific detonation time, when a projectile penetrates into the multilayer medium.

The projectile/target model uses several assumptions. It is assumed that the attack angle of the projectile is not considered here; the mass of explosive is constant; the detonation time is known as penetration time, and it is set to 1.2ms; the explosion effect is also not considered after the detonation time. Hence, penetration depth, which is described as the vertical displacement between the foreside of the projectile and the original top surface of multilayer medium, can be measured based on the numerical results by a forward solver. The state angle, which is described as deviation angle between axes of the projectile and the vertical of the multilayer medium, can be also measured. In general, the typical projectile is a kinetic energy bomb, and the typical multilayer medium consists of concrete layer, gravel layer and soil layer, as illustrated in Fig. 1. In this model, thickness of concrete layer, gravel layer and soil layer is $a = 0.2m$, $b = 0.25m and c = 0.3m$, respectively. The projectile geometry consists of a 3.0 CRH ogival nose with a total length 0.032m, and an aft body diameter 0.006m.

In this study, the unknown input vector **X** includes striking velocity (v) and oblique angle (α) of the projectile. These parameters are initially left unspecified and will then determined by using the present inverse technique. **Y**denotes the known output vector which includes penetration depth (d) and state angle (β). It obtained

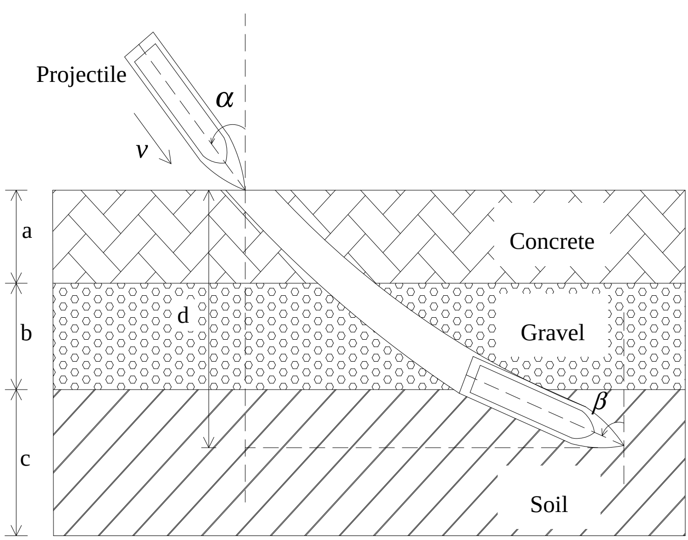


Figure 1: Geometry of the projectile/target system.

from the simulated experiment measures, and obeys probability distribution with a certain mean value and variance. Because of the uncertainty propagation, it is clear that the unknown encounter condition parameters in the input **X** will also follow a certain probability distribution.

3 Bayesian identification of the encounter condition

The general Bayesian framework for determining the encounter condition and the MCMC method based on an approximation model strategy are discussed in this section.

3.1 Bayesian approach

Mathematically, Bayesian identification for the encounter condition in the projectile/target system can be expressed as [Sivia and Skilling (2006)]

$$P(\mathbf{X}|\mathbf{Y}) = P(\mathbf{Y}|\mathbf{X})P(\mathbf{X})/P(\mathbf{Y}) \tag{1}$$

where **Y** and **X**are observed projectile state data vector and unknown encounter condition vector, respectively. $P(\mathbf{X})$ denotes the prior probability density for the encounter condition parameters. $P(\mathbf{Y}|\mathbf{X})$ denotes the data likelihood, and can be regarded as a measure of how well we fit the observed projectile state data. $P(\mathbf{X}|\mathbf{Y})$ denotes the posterior probability density function (PPDF) of the unknown encounter condition parameters. In fact, mechanism of the Bayesian approach is that the prior knowledge $P(\mathbf{X})$ is updated with the data likelihood $P(\mathbf{Y}|\mathbf{X})$ to achieve the current state of knowledge, the PPDF $P(\mathbf{X}|\mathbf{Y})$. $P(\mathbf{Y})$ is a normalizing constant that can be ignored in the following development, and Eq. (1) can be rewritten as

$$P(\mathbf{X}|\mathbf{Y}) \propto P(\mathbf{Y}|\mathbf{X})P(\mathbf{X}) \tag{2}$$

Assume that we have measured a projectile state **Y** at a specific detonation time with noise

$$y_n = y_n + \eta_n,\ n = 1, 2, \cdots, N \tag{3}$$

where d_n is the noise-free response and η_n a sequence of independent, identically distributed samples drawn from a zero-mean Gaussian distribution with variance σ^2. In our model, number of the observed variable is set to 2. In this situation, the likelihood can be rewritten as

$$P(\mathbf{Y}|\mathbf{X}) = P_L(\mathbf{Y}|\mathbf{X}, \sigma^2) = \frac{1}{2\pi\sigma^2}\exp(-\frac{1}{2\sigma^2}\sum_{n=1}^{2}(y_n - f_n(\mathbf{X}))^2) \tag{4}$$

where$f_n(\mathbf{X})$ is the projectile state response obtained from a forward analysis.

Additionally, the prior knowledge is usually treated as a uniform distribution due to its fine ability of integrating the practical experience and experts' knowledge for the practical engineering. Thus, $P(\mathbf{X})$ is often regarded as a constant in the given prior parameter knowledge space, and Eq. (2) can be finally rewritten as

$$P(\mathbf{X}|\mathbf{Y}) \propto P_L(\mathbf{Y}|\mathbf{X},\sigma^2)P(\mathbf{X}) = r\cdot\exp\left(-\frac{1}{2\sigma^2}\sum_{n=1}^{2}(d_n - f_n(\mathbf{X}))^2\right) \tag{5}$$

where $P_J(\mathbf{X}|\mathbf{Y})$ is the joint posterior distribution for the encounter condition parameters. r represents a constant which has not influence on the identified results.

The form of Bayesian identification for the encounter condition given in Eq. (5) is for the joint parameter distribution whereas we are typically interested in marginal PPDF of each encounter condition parameter. Theoretically, this would require integrating Eq. (5) over the other $M-1$ parameters

$$P(\mathbf{X}_i|\mathbf{Y}) = \int_{R^{M-1}} P_f(\mathbf{X}|\mathbf{Y})d\mathbf{X}_{-i} \propto \int_{R^{M-1}} r\cdot\exp\left(-\frac{1}{2\sigma^2}\sum_{n=1}^{2}(d_n - f_n(\mathbf{X}))^2\right)d\mathbf{X}_{-i} \tag{6}$$

where $P_H(X_i|\mathbf{Y})$ represents the marginal PPDF. $\int_{R^{M-1}} P(\mathbf{X}|\mathbf{Y})d\mathbf{X}_{-i}$ denotes the multidimensional integral over all parameters other than m_{-i}. M is number of the unknown encounter condition variable.

Solving Eq. (6) is a key step in the present Bayesian identification of the encounter condition. Whereafter, it can provide mean value of the posterior and construct confidence intervals for each of the encounter condition parameters. Thus, we need an effective method to solve Eq. (6) involving a complicated likelihood function in this problem.

3.2 The traditional MCMC

Among various algorithms to solve Eq. (6), the MCMC method appears more versatile and powerful. The MCMC was first proposed by [Hastings (1970)] as a means of drawing samples from the PPDF directly using a Markov Chain without having to integrate. Its idea is to generate a stationary Markov chain for each encounter condition parameter in $\mathbf{X}$ such that the values in the chain are samples from the unknown individual posterior distribution. The approach works as follows. For the ith parameter, the chain is started at a initial value X_i^0 chosen randomly from the prior distribution $P(X_i)$. The next value in the chain is formed by first proposing state transitions for each parameter using a so called proposal distribution $q(X_i^*|X_i)$. This distribution provides a rule for generating a candidate parameter value X_i^* given the current value in the chain X_i. The proposed parameter (keeping all other parameters fixed at their current value) is then accepted or rejected with probability

$$r = \max\left(\frac{P_{J_i}(X_i^*|\mathbf{Y})q(X_i^*|X_i^{k-1})}{P_{J_i}(X_i^{k-1}|\mathbf{Y})q(X_i^{k-1}|X_i^*)}, 1\right) \tag{7}$$

If the trial succeeds, the perturbed parameter value become the next value in the chain $X_i^k = X_i$, otherwise the previous value is retained. The same is done for each of the encounter condition parameter while holding the other $M-1$ parameters fixed. Thus, the approach is really drawing samples from the marginal PPDF $P_H(x_i|\mathbf{Y})$. Additionally, if either the prior or proposal distribution is uniform, then they cancel in the ratio. So if both the prior and proposal distributions are uniform, and then r is simply a ratio of the likelihoods. It should be noted that the proposal distribution is theoretically an arbitrary probability distribution, but a proper distribution selection leads to much better convergence in the resulting Markov chain.

The formation of the Markov Chain plays a key role in this algorithm. Repeating the above process a huge number of times can generate a chain that forms a marginal PPDF for the true value of the each encounter condition parameter. The chains evolve until convergence to a stationary distribution is achieved. Therefore, the chain length commonly required o (10^5) samples, making the method very time-consuming especially for this complex projectile/target model.

3.3 A MCMC method based on the approximation model

To improve the efficiency of the Bayesian approach, the special attention is paid to developing a fast and accurate approximation model for the response of the projectile/target system. Since it has to be carried out many times, from Eq. (6), we can easily found that the $f_n(\mathbf{X})$ in the PPDF plays a key effect on computational cost for the Bayesian identification. Thus, the computational efficiency can be remarkably improved by constructing the approximate $\bar{f}_n(\mathbf{X})$ based on an approximation model method.

The basic idea of the present method is first use an approximation model to probe the entire prior knowledge space quickly and efficiently to approximately determine joint PPDF space. Then, use the resulting cheap-to-evaluate joint PPDF model in the conventional MCMC, and the marginal PPDF for each encounter condition parameter can be rapidly obtained without sacrificing accuracy.

There are several kinds of approximation methods Mysers and Montgomery (2002): kriging model, radial basis function model and polynomial response surface method, etc. In this work, the polynomial response surface based on structure-selection techniques is adopted to substitute the actual FEM projectile/target model due to its fine performance on computational efficiency, numerical stability and capacity of capturing nonlinear behavior. The basic idea of this method is to select the effective parameter of the polynomial model by using a criterion called error-reduction-ratio. This technique is now fairly standard and the reader is referred to [Lu at el. (2005)] for a detailed description. An experiment design method [Peter and Bradley (2011)] is performed with the computationally expensive model f_n to create a data set of the unknown encounter condition parameters $\mathbf{X}$ and corresponding projectile state responses $\bar{f}_n(\mathbf{X})$. This data set is then used to train the approximation model to receive $\mathbf{X}$ as input and produce an approximation to the response of the $\bar{f}_n(\mathbf{X})$ as output. Thus, the form of Bayesian approach Eq. (6) base on the approximation model can be written as

$$\overline{P_H}(X_i|Y) \propto \int_{R^{M-1}} r \cdot \exp\left(-\frac{1}{2\sigma^2} \cdot \sum_{n=1}^{2} (d_n - \bar{f}_n(\mathbf{X}))^2\right) d\mathbf{X}_{-i} \tag{8}$$

where $\overline{P_H}(X_i|\mathbf{Y})$ denotes the approximate marginal PPDF. $\overline{f_n}(\mathbf{X})$ denotes the approximate $f(\mathbf{X})$.

Apparently, $\overline{P_H}(X_i|\mathbf{Y})$ in Eq. (8) can be rapidly constructed to provide approximation to the marginal PPDF for the encounter condition parameters $P_H(X_i|\mathbf{Y})$ with very few expensive forward calculations. In addition, since using the cheap model $\overline{f_n}(\mathbf{X})$, the computational burden in the conventional MCMC can be successfully reduced. Therefore, the marginal PPDF for each encounter condition parameter can be rapidly obtained.

It can be found that the computational cost is spent in two parts. The first part is to calculate the set of samples data by using a forward solver for constructing the joint PPDF approximation model. For this complex problem, it would dominate the process. The second one is to determine the means and the confidence intervals for each encounter condition parameter by using the traditional MCMC. As the approximation model is used, the computational efficiency can be very high in this process. The forward analysis and the construction of the approximation model are discussion in the following section.

4 Forward analysis

4.1 Finite element model

In the forward calculation, we need to calculate the penetration depth and state angle for the trial striking velocity and oblique angle of the projectile. Accuracy is very important, as the process of the projectile penetrating into multilayer medium is very complex that associated with high-velocity penetration, large deformation and wave propagation, etc. Therefore, LS-DYNA is used as the forward solver to calculate these cases due to its outstanding accuracy for penetration problems.

The projectile loses small amounts of mass through abrasion and experienced relatively smaller deformations during the penetration process. Thus, metal shell of the projectile is considered as rigid-body. To take into account effect of the large strains, high strain rates and high pressures of the concrete, the Holmquist-Johnson-Cook (HJC) model is adopted. This constitutive model, which is proposed by [Holmquist et al. (1993)], is widely used to describe the dynamic characteristic of the concrete. Nevertheless, how to describe the dynamic behavior of the gravel is really a complex subject, and which is not the core in this paper. Hence, in order to reduce the difficulty of constructing the gravel constitutive relationship, we make use of the equivalent concrete model to approximately describe the gravel model in the allowable range fro the engineering design. Four equivalent parameters such as shear modulus G, Quasi-static uniaxial compressive strengthf_c', Maximum tensile hydrostatic pressure T and crushing pressure p_{crush} are different from the concrete constitutive model. Moreover, the soil is modeled with "soil_and_foam" material model which is very simple, and widely adopted to describe the dynamic characteristic of soil. Detailed descriptions of above material models can be found in LS-DYNA theoretical manual [Hallquist et al. (2006)].

Due to the symmetric characteristic, only half of the projectile/target system is modeled in this work. The concrete, gravel and soil are modeled with eight nodes uniform hexahedron solid elements whilst the projectile is modeled with six nodes tetrahedron solid elements. The nodes making up the projectile mesh are assigned a striking velocity. The projectile/target system is totally discretized into 503595 elements. The radius dimension of the elements increased with the radius, and the original mesh for a normal penetration is shown in Fig. 2.

The boundary conditions are specified as follows: the top surface of the target is free face. The lower, left, and right sides are non-reflecting boundaries, and the front one is symmetric boundary. The contact element is used to connect the concrete, gravel and soil layers. This element allows the application of the maximum normal tensile stress failure criterion. The erosion algorithm is used to deal with the excessive element distortion problem. Hence, the calculation can be carried out without the need for re-zoning distorted regions of the mesh during the penetration process.

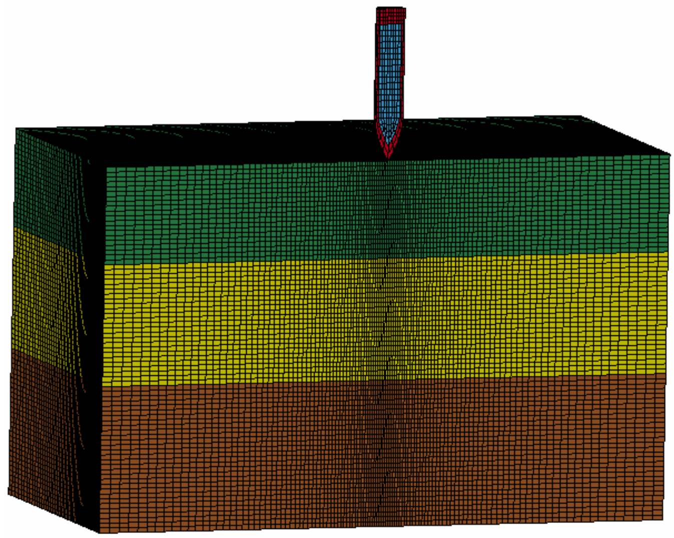
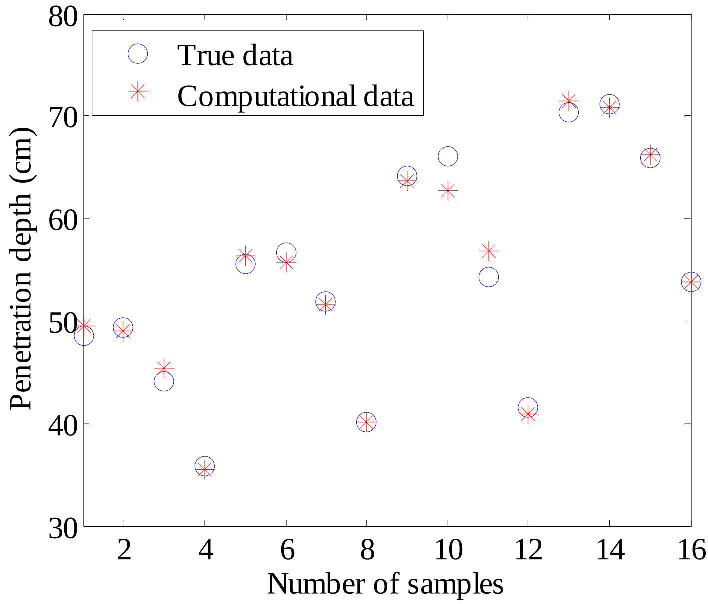


Figure 2: 3D FEM mesh of the projectile/target model.

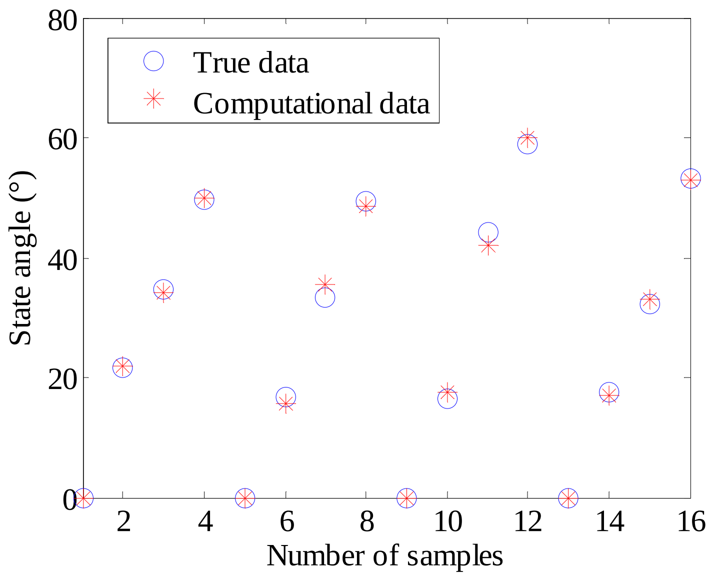
4.2 Approximation model

There are several different experimental design methods [Peter and Bradley (2011)] available, such as the full factorial, Latin Hypercube and D-optimal design, etc. Here, we made use of the full factorial method for its uniformity of sampling. Specifically, a k^s full factorial experimental design generates a mesh of sample consisting of $k$points spaced at regular intervals in each variable direction. A four-level full factorial design is used for sampling, which resulted in16 ($k = 4, s = 2$) evenly distributed samples in the range of the encounter condition. The range of the striking velocity and oblique angle of the projectile is $v \in [520m/s,$ 720m / s] and $\alpha \in [0°, 45°]$, respectively. The 16 simulations are run on personal computer, which is an eight 2.81 GHz Intel Core i7 processor with 4 GB of RAM running Windows XP.

Using the results of the 16 calculations, the approximation model of penetration depth and state angle are both constructed by using the polynomial response surface based on the structure-selection technique. Comparison of projectile state obtained by the approximation model and true data are shown in Fig. 3. It can be observed that the fit of the approximation model to the true data could be considered reasonable. In order to better explain, the relative error between the computational data and the true ones are depicted in Fig. 4. We can easily found that the maximum value of the relative error is no more than 7% which occur in No. 6 sample in



(a)



(b)

Figure 3: Verification of the approximation model, (a) verification of the penetration depth and (b) verification of the state angle.

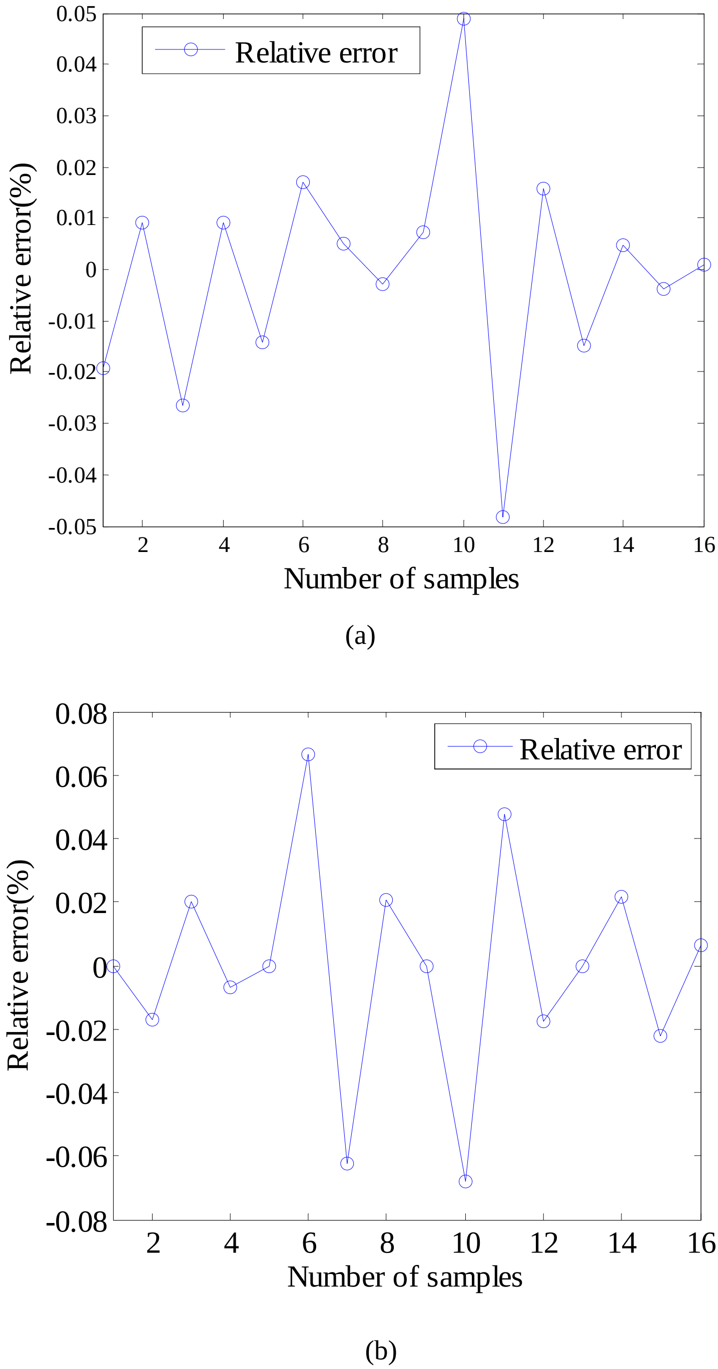


Figure 4: Relative error of the approximation model, (a) relative error of the penetration depth and (b) relative error of the state angle.

Fig. 4(b). It can be concluded from Fig. (3) and (4) that the approximation model has a fine accuracy, and it can be used as an approximate forward solver in the subsequently inverse procedure.

5 Results and discussion

In this section, two numerical cases are given to test the effectiveness of the proposed method. The following experimental procedure is simulated. The required experimental projectile state **Y** is a FEM simulation result for the projectile/target system with known encounter condition parameters **X** along with a specified level of Gaussian noise superposed.

5.1 Case 1

The initial prior knowledge of the encounter condition **X** is given as a uniform distribution which is described by $v \sim U(520m\,/\,s, 720m\,/\,s)$ and $\sigma \sim U(0^\circ, 45^\circ)$. 20 000 iterations are run in the identification process. The true encounter condition parameters are v_{true} = $636.2m\,/\,s$ and α_{true} = 23.4°. The corresponding the mean of the projectile state parameters are d_{mean} = 0.59m and β_{mean} = 28.1°. Several noise levels of the projectile state are considered in this case.

Firstly, we illustrate the speedup achieved by the approximation model. The approximation model of the projectile/target system can be performed off-line in order to achieve online computational efficiency. Upon obtaining the approximation model, its calculation is very cheap. The initial construction cost is about 240 h for 16 calculations of carrying the real FEM model, and thereafter the computing time is rapidly eclipsed by the direct calculation. For this case, the approximation model takes about 13.5 seconds to complete the all calculations by using the MCMC, whereas the real FEM model takes about 15 hours for only a single calculation. Therefore, it can be found that the speedup over direct calculation is quite dramatic for this complex projectile/target model.

Next, we demonstrate the usage of the MCMC to sample the posterior state space based on the approximation model. Fig. 5 shows the 20 000 samples in the 2D parametric plane (v, α). The 2D view describes the main feature of the posterior density defined in Eq. (5). In the MCMC process proceeds, the encounter condition parameters, striking velocity v and oblique angle α are all perturbed in turn and updated to their new values as displayed in Fig. 6. This shows each of the individual encounter condition parameters as a function of iteration number. The true value (associated with Y) is given by the horizontal line. It can be found that the MCMC chain is stationary and the perturbations continue in order to generate the parameter distributions. In fact, the output of these stationary chains is a marginal PPDF for

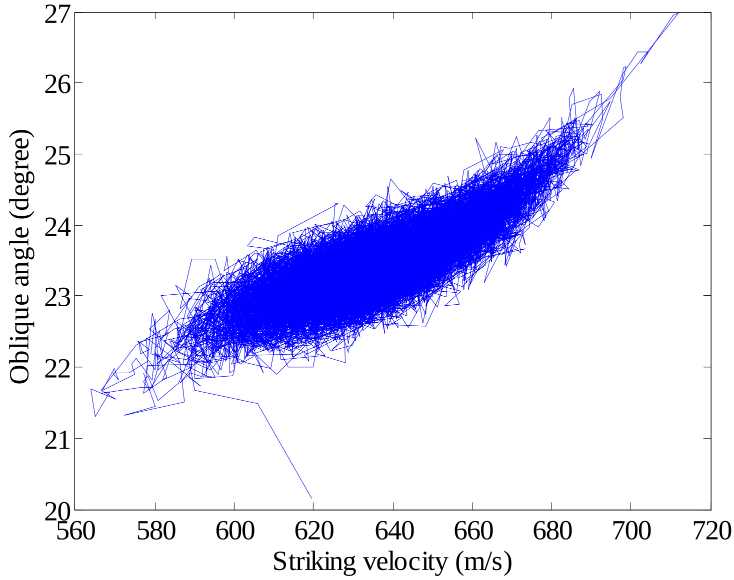


Figure 5: 2D trace plot of the Markov chain for case 1 with 3% noise added in the projectile state data.

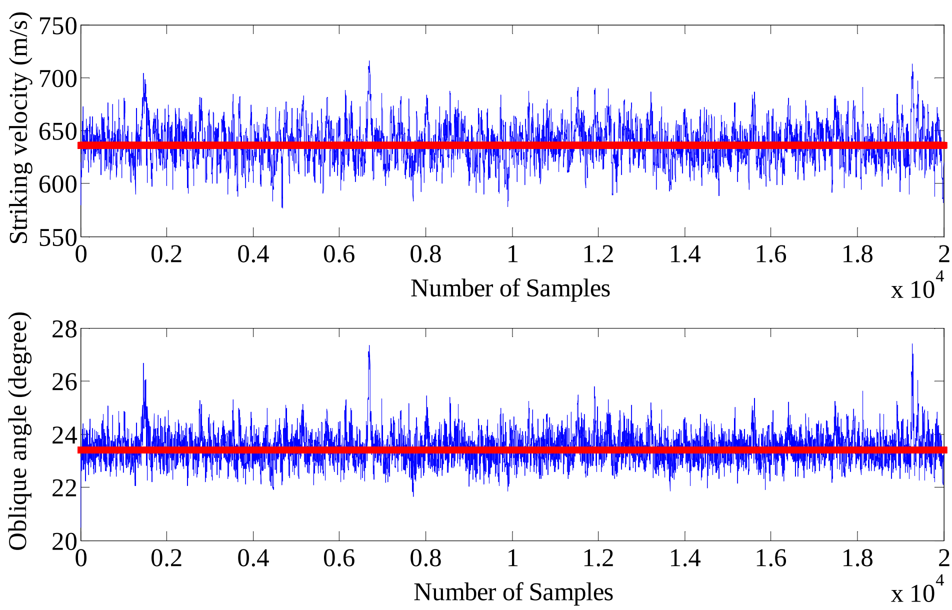
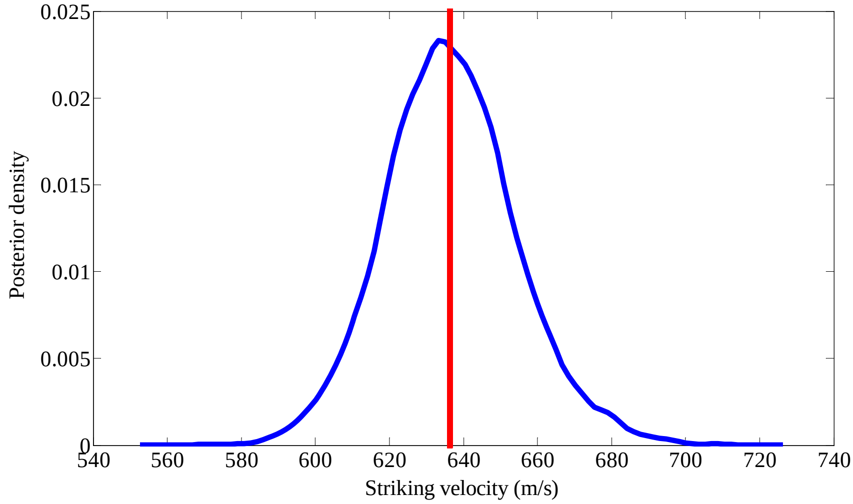
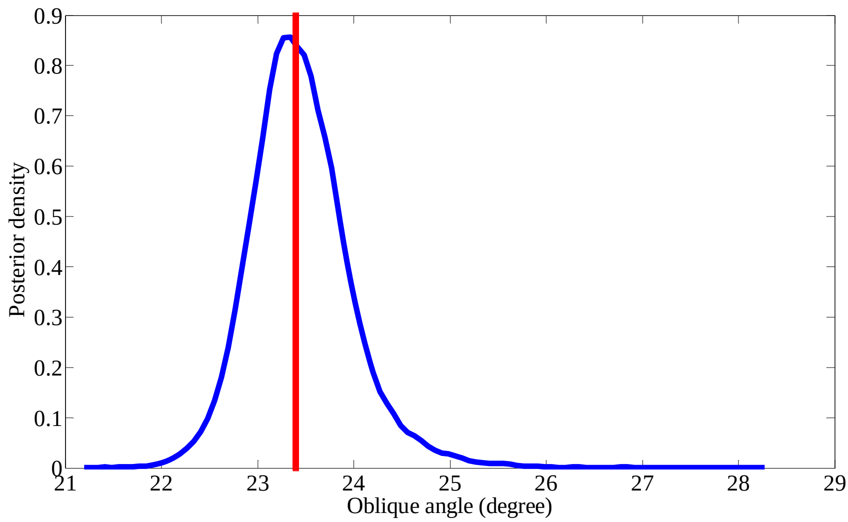


Figure 6: 1D trace plot of the Markov chain for case 1.



(a)



(b)

Figure 7: Posterior density of the encounter condition parameters resulting obtained by the MCMC, (a) striking velocity and (b) oblique angle.

each encounter condition parameter. The associated marginal PPDFs are shown in Fig. 7. The vertical line in each PPDF represents the true value of the parameter. Noting the abscissa axis of Fig. 7, it is evident that the present inverse technique provides very reasonable estimates of the encounter condition parameters, at least for this one case. Their associated distributions are also quite narrow, suggesting a high degree of confidence in the results.

The convergence curves for the means of the each encounter condition parameter are shown in Fig. 8. The true values of the encounter condition parameters are given by the horizontal line. We found that the fitness of the identified results and the true ones is very good after complete about 3 000 iterations. Therefore, it appears that the sample size of 20 000 is more than sufficient to give good precision for the posterior mean estimate.

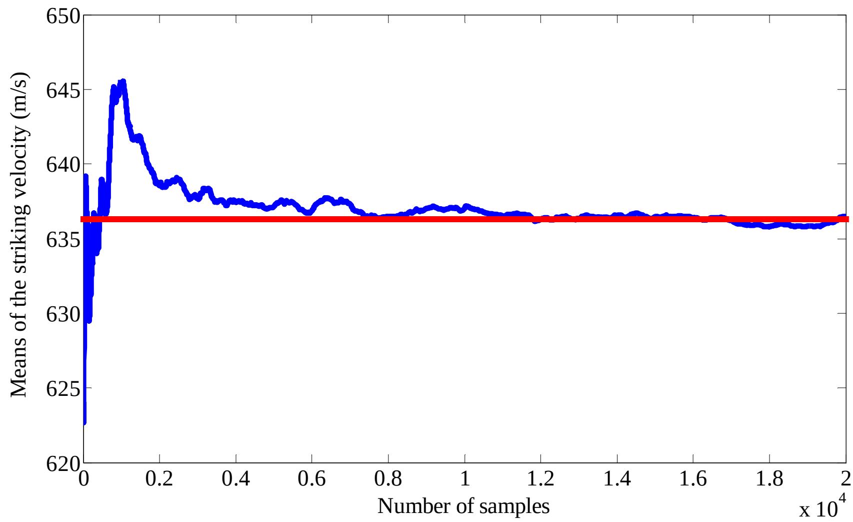
Finally, Fig. 9 shows the 95% confidence intervals, defined as the central 95% of the 20 000 values that comprise the stationary Markov chain, as a function of noise level. As expected, the confidence intervals tend to wide as the noise level is increased. Furthermore, it can be seen that each of the encounter condition parameters is correctly identified with their associated confidence intervals using the present technique, at least in the low noise level. Let us use resulting striking velocity v with 3% noise added as an example to explain what the mean of the obtained results are. The result can be stated as the following: there is a 95 percent chance that the striking velocity v lies in the interval $601.47m/s \le v \le 671.46m/s$, and mean is $v_{mean} = 636.47m/s$.

The above results have demonstrated that the means of the encounter condition parameters and their confidence intervals can be rapidly and successfully identified from the projectile state data with measurement noise by using the present inverse technique. Furthermore, the propagation of the uncertainty from the projectile state to the identified encounter condition is accurately quantified, which has a great value for practical engineering applications.

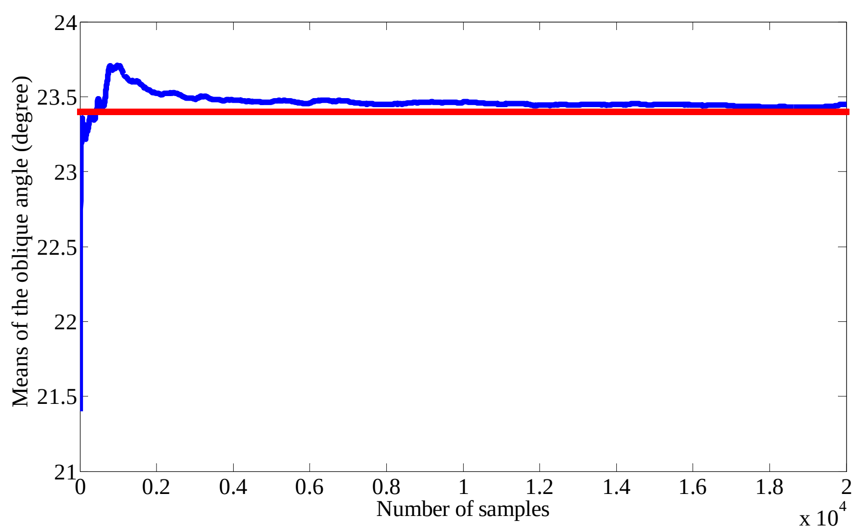
5.2 Case 2

As a second case, we seek to identify the encounter condition using the other set of the projectile state data. The true encounter condition parameters are $v_{true} = 590m/s$ and $\alpha_{true} = 15°$. The corresponding the mean of the projectile state parameters are $d_{mean} = 0.50$m and $\beta_{mean} = 35.8°$. The 5% noise of the projectile state is considered in this case. This case can be solved in the exactly same way as the case 1.

Firstly, 3D view for the true joint PPDF of the unknown encounter condition parameters is plotted in Fig. 10. It provides us a tool to compare the resulting the marginal PPDF and the true joint PPDF for the obtained encounter condition pa-

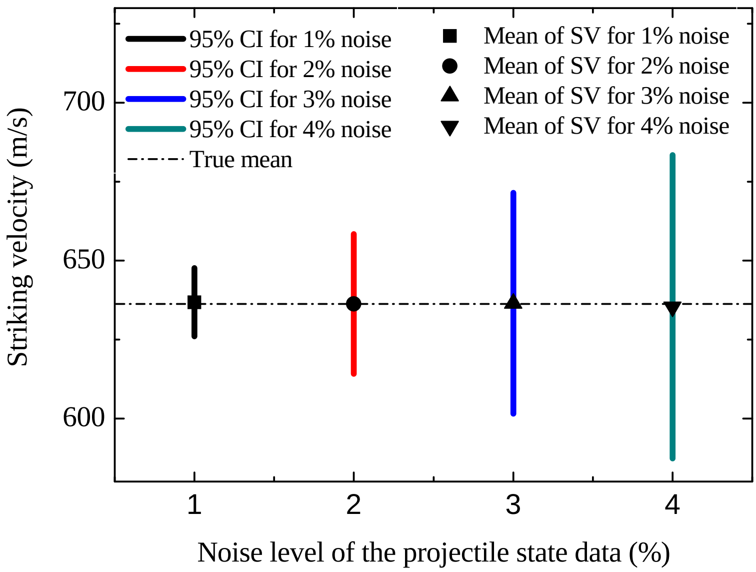


(a)

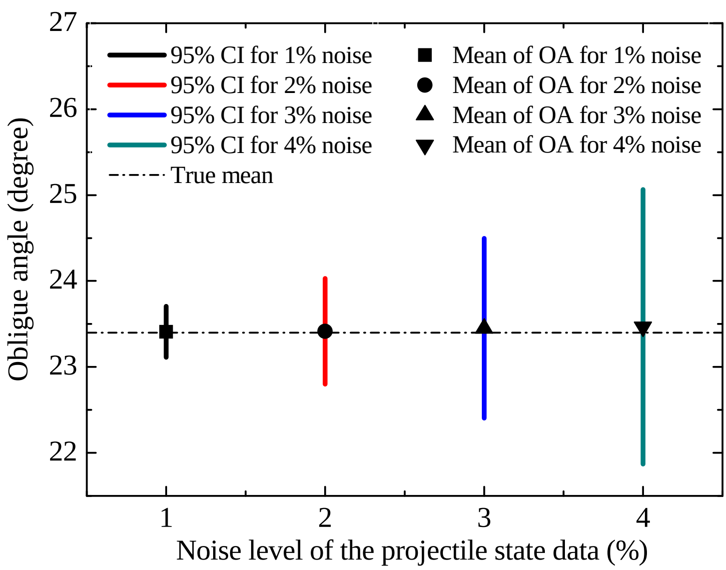


(b)

Figure 8: The convergence curves for the means of the encounter condition parameters obtained from the 3% noise added projectile state data, (a) striking velocity and (b) oblique angle.



(a)



(b)

Figure 9: 95% confidence intervals for the encounter condition parameter estimates as a function of noise level, (a) striking velocity and (b) oblique angle.

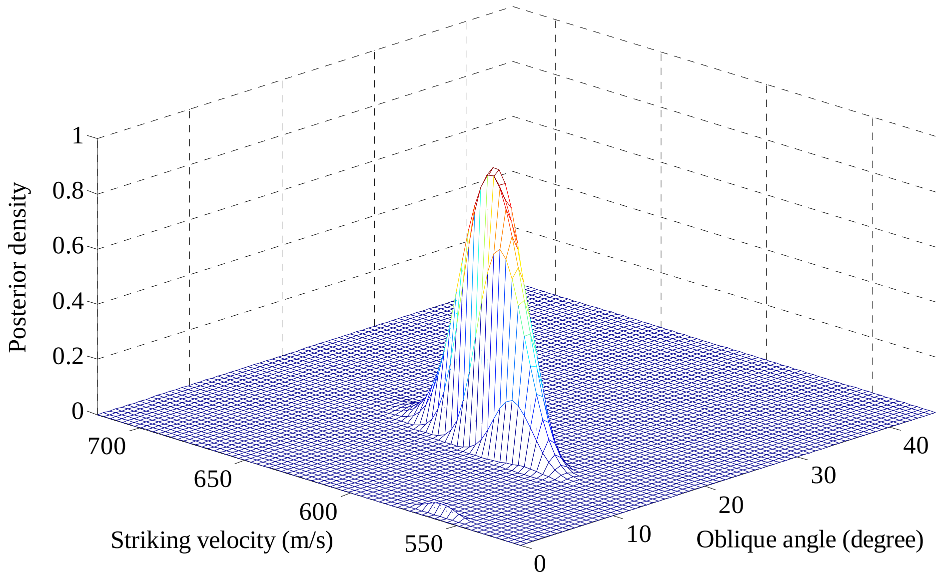


Figure 10: 3D view of the joint PPDF for case 2 with 5% noise added in the projectile state data.

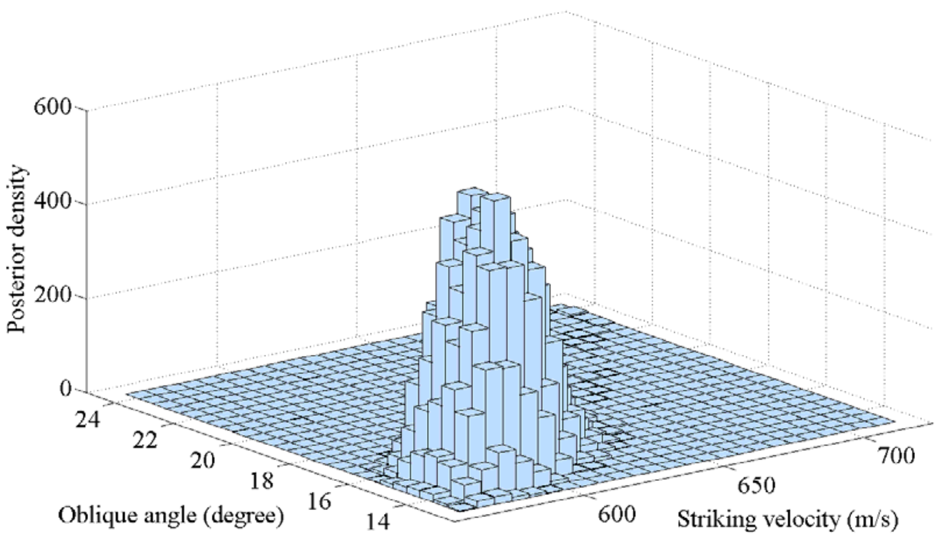


Figure 11: 3D histogram of the joint PPDF for case 2 obtained by the present technique.

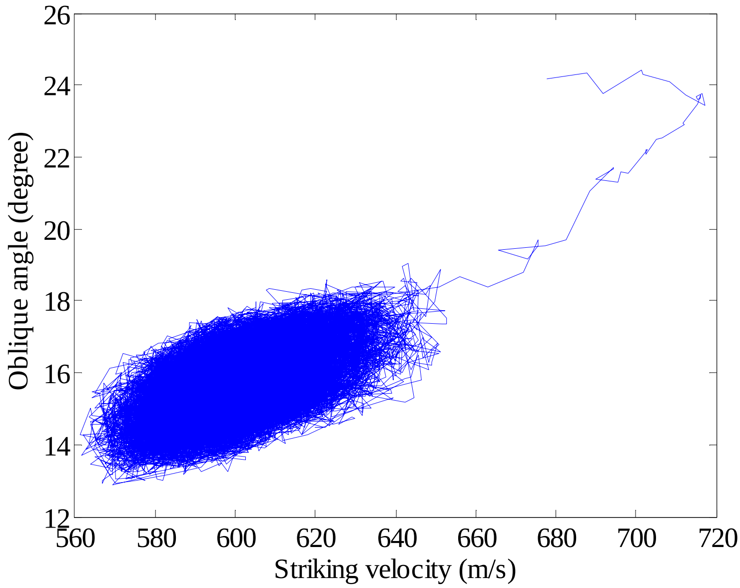


Figure 12: 2D trace plot of the Markov chain for case 2.

rameters. Additionally, the visualization of the joint PPDF of the parameters helps us to further understand the identification process.

A 3D histogram is used to summarize the marginal distribution properties of the posterior samples, as shown in Fig. 11. It can be found that the histogram view is reminiscent of the 3D view shown in Fig. 10. Thus, it can be also concluded that the proposed method has a fine accuracy to identify the encounter condition parameters.

Secondly, the usage of the MCMC method to sample the approximate joint PPDF is also studied. Fig. 12 shows the 20 000 samples in the 2D parametric plane (v, α). The 2D view describes the projection of the posterior density onto the parameter axis plane. In order to show the mixing of the chain better, the individual trace plot of v and α over 20 000 iterations is shown in Fig. 13. The true value (associated with Y) is given by the horizontal line. It can be also found that the Markov Chains are stationary and the perturbations continue in order to generate the reliable parameter distributions in this case.

Finally, Fig. 14 shows the identified posterior distributions for this case where the true encounter condition parameters are set to the values $v = 590m/s$ and $\alpha = 15°$. It is clearly seen that the uniform prior is translated to the posterior by using the proposed technique. Moreover, all encounter condition parameters are again correctly

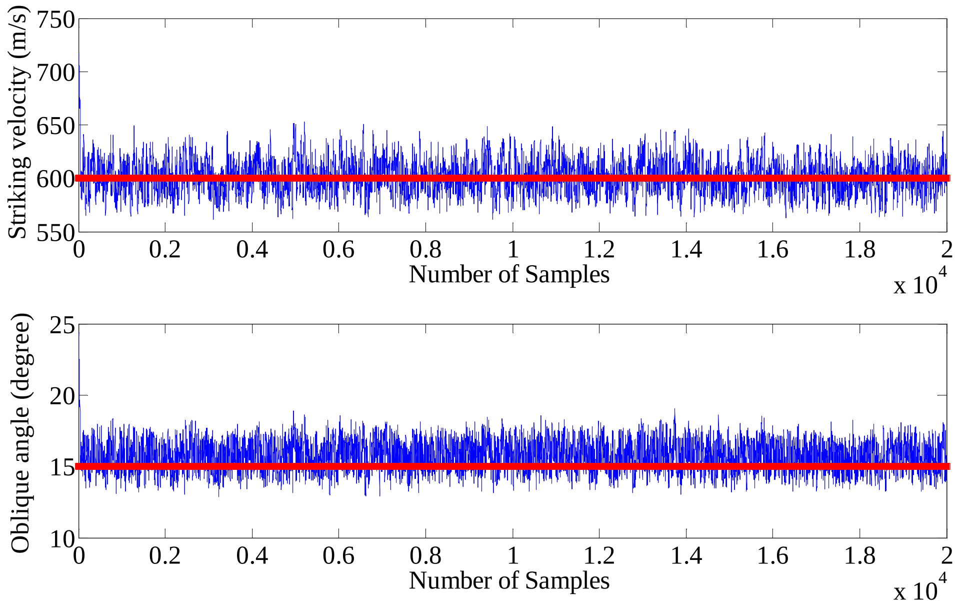
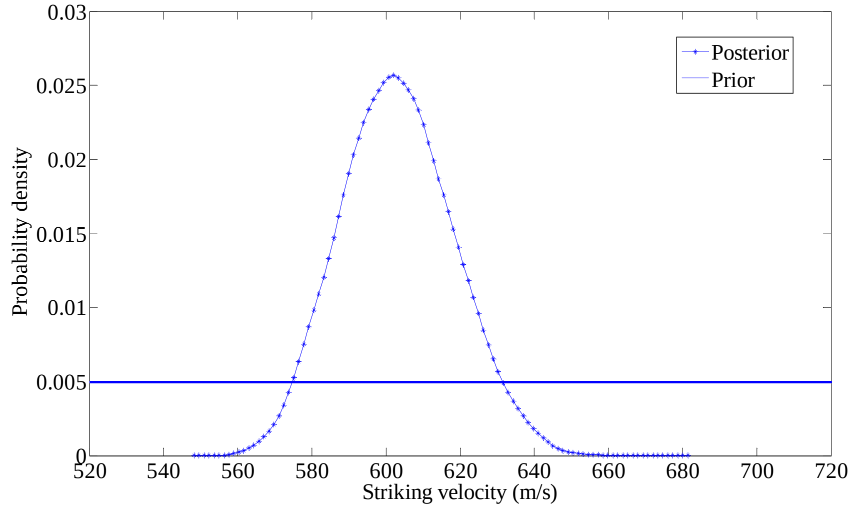


Figure 13: 1D trace plot of the Markov chain for case 2.

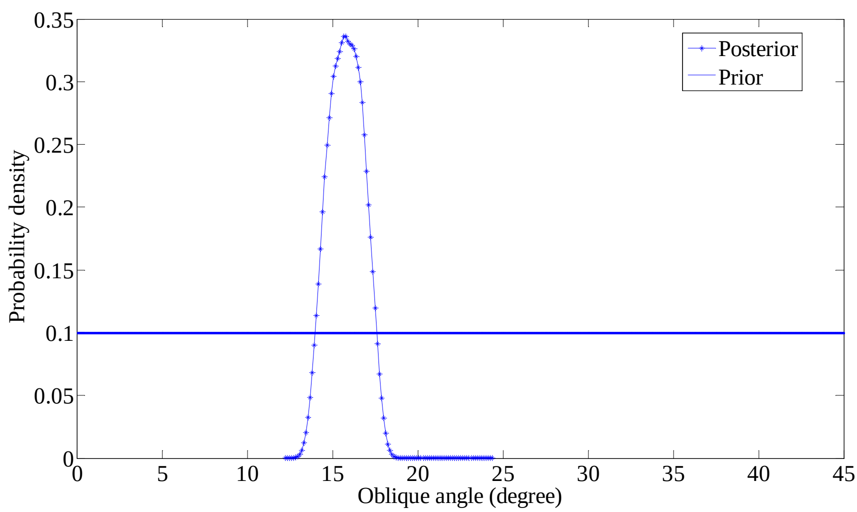
identified, provided that the final estimates are taken as the maximum a posterior value and their distributions. We can easily found from Fig. (14) that the results can help us to make an optimal decision to use these identified encounter condition parameters in the further numerical analysis or engineering design.

6 Conclusions

In this paper, a computational inverse technique combined the approximation model method and Bayesian approach is proposed for quantifying the uncertainty in an encounter condition identification problem. This identification process is then implemented on simulated data. When combined with the prior knowledge, this technique can rapidly and accurately provide the information necessary for helping us to identify the encounter condition parameters and assess their reliability. We illustrate the applications of the present technique with two numerical examples. The results clearly indicate that this technique holds real promise for successfully identifying the encounter condition in the practical engineering. Moreover, the proposed technique is not limited to the projectile/target system but can be easily applied to the other engineering fields, e.g. heat source identification, load reconstruction and model calibration.



(a)



(b)

Figure 14: Estimated PPDFs for each encounter condition parameter, (a) striking velocity and (b) oblique angle. The true value $v = 600m/s$, $\sigma = 15°$.

Acknowledgement: This work is supported by the General Program of National Natural Science Foundation of China (51175160) and the Key Project of Chinese National Programs for Fundamental Research and Development (2010CB832700).

References

Forrestal, M. J.; Frew, D. J.; Hickerson, J. P.; Rohwer, T. A. (2003): Penetration of concrete targets with deceleration-time measurements. *Int. J. Impact. Eng*, vol. 28, pp. 479-497.

Gawin, D.; Pesavento, F.; Schrefler, B. A. (2011): What physical phenomena can be neglected when modelling concrete at high temperature? A comparative study, part 1: physical phenomena and mathematical model. *Int. J. Solids. Struct*, vol. 48, pp. 1927-1944.

Goldsmith, W. (1999): Non-ideal projectile impact on targets. *Int. J. Impact. Eng*, vol. 22, pp. 95-395.

Hallquist, J. Q. (2006): LS-DYNA theoretical manual, Livermore Software Technology Corporation, Livermore.

Hastings, W. K. (1970): Monte carlo sampling methods using markov chains and their applications. *Biometrika*, vol. 57, pp. 97-109.

Homquist, T. J.; Johnson, G. R.; Cook, W. H. (1993): A computational constitutive model for concrete subjected to large strains, high strain rates, and high pressure. Fourteenth International Symposium on Ballistics, Quebec, 1-10.

Li, Q. M.; Chen, X. W. (2003): Dimensionless formulae for penetration depth of concrete target impacted by a non-deformable projectile. *Int. J. Impact. Eng*, vol. 28, pp. 93-116.

Lu, F. F.; Xu, D. L.; Wen, G. L. (2005): Tracing initial condition, historical evolutionary path and parameters of chaotic process from a short segment of scalar time series. *Chaos, Solitons. Fract*, vol. 24, pp. 265-271.

Marin, E. B.; Chiesa, M. L.; Booker, P. M. (2006): Parametric studies of penetration events: a design and analysis of experiments approach. Technical Report SAND20050951, Sandia National Laboratories, Albuquerque, New Mexico.

Moore, E.; Murphy, K.; Nichols, J. (2011): Crack identification in a freely vibrating plate using Bayesian parameter estimation. *Mech. Syst. Signal. Pr*, vol. 25, pp. 2125-2134.

Mysers, R. H.; Montgomery, D. C. (2002): Response surface methodology: process and product optimization using designed experiments, Wiley, New York.

Nichols, J.; Moore, E.; Murphy, K. (2011): Bayesian identification of a cracked plate using a population-based Markov Chain Carlo Method. *Comput. Struct*, vol.

89, pp. 1323-1332.

Peter, G.; Bradley, J. (2011): Optimal design of experiments: a case study approach, Wiley, New York.

Sivia, D.; Skilling, J. (2006): Data analysis: a Bayesian tutorial, Oxford, London.

Yan, L.; Yang, F.; Fu, C. (2011): A new numerical method for the inverse source problem from a Bayesian perspective. *Int. J. Numer. Meth. Eng*, vol. 85, pp. 1460-1474.

Zhang, E.; J. Feissel, A. (2012): Bayesian force reconstruction with an uncertain model. *J. Sound. Vib*, vol. 331, pp. 798-814.

Zhang, W.; Han, X.; Liu, J.; Ning, H. (2012a): A Fast Bayesian approach for parameter identifications using adaptive densifying approximation technique. *Comput. Struct*. (Under review).

Zhang, W.; Liu, J.; Han, X.; Chen, R. (2012b): A computational inverse technique for determination of encounter condition on projectile penetrating multilayer medium. *Inverse. Probl. Sci. En*, (on line), DOI: 10.1080/ 17415977. 2012. 659733.