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Evolutionary Algorithms Applied to Estimation of Thermal Property by Inverse Problem

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Abstract: In this study an inverse heat conduction problem using two optimization methods to estimate apparent thermal diffusivity at different drying temperatures is solved. Temperature and moisture versus time were obtained numerically using heat and mass transfer equations with drying temperatures in the range between 20°C to 70°C. The solution of the partial differential equation is made with a finite difference method coupled to optimization techniques of Differential Evolution (DE) and Particle Swarm Optimization (PSO) used in inverse problem. Statistical analysis shows no significant differences between reported and estimated curves, and no remarkable differences between results obtained using DE and PSO in 30 runs. The convective and evaporative effects and shrinkage assumptions in the model provides greater reliability on the calculated thermal diffusivity.

Keywords: Inverse problem, Thermal diffusivity, Optimization, Differential evolution, Particle swarm optimization, Finite difference method.

1 Introduction

There are numerous methods to measure thermal diffusivity proposed in the specialized literature. Nevertheless, most of them need relatively complex instrumentation or experimental assemblies and demand an expertise of the thermal phenomena. Sweat (1986) recommends determination of thermal diffusivities from experimentally obtained values for thermal conductivity, specific heat and mass density.

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Growing interest has recently been evidenced in the analysis and solution of inverse problems in different areas, and some study specifically obtaining thermal properties. For example, Liu et al. (2007) identify thermophysical parameters of the inverse heat conduction problems governed by linear parabolic partial differential equations (PDEs) establishing one-step group preserving scheme for the semi-discretization of PDEs. Liu and Atluri (2008) developed a fictitious time integration method (FTIM) to find the potential function, impedance function or weighting function, in a discretized manner by inverse problem. Liu and Atluri (2008) solve the inverse Sturm-Liouville problem applying Lie-group and FTIM methods, with significant improved accuracy. Liu (2008) consider a Lavrentiev regularization and Fredholm integral equation to solve an inverse problem of Laplace, Cauchy, as well as the problem of unknown Robin coefficient. The Calderón inverse problem is reduced to an inverse Cauchy and parameter identification problem in Liu and Atluri (2010). Mendonça et al. (2005) and Simpson and Cortés (2004) using the inverse method to estimate thermophysical properties of foods.

The meshless local Petrov-Galerkin method is used to solve the inverse heat conduction problem predicting the distribution of the heat transfer coefficient on the boundary of bidimensional and axisymmetric bodies in Sladek et al. (2009). The method of fundamental solutions is coupled with the boundary control technique to solve the Cauchy problems of the Laplace equations by Ling and Takeuchi (2008).

An inverse forced vibration problem is studied in Huang and Shih (2007) to estimate the unknown time-dependent applied force and moment for an Euler-Bernoulli beam. Marin et al. (2008) investigate the reconstruction of a divergence-free surface current distribution in the framework of static electromagnetism. Solvability conditions of an inverse problem for non-stationary kinetic equation is formulated by Yildiz (2009).

Many papers reported in the literature involving inverse problems use deterministic methods, based on gradient information, to minimize the objective function (Khachf et al., 2002). Although such optimization methods can lead to local rather than global minima, their main advantage lies in their good convergence rate. New optimization methodologies are being used to solve inverse problems, particularly stochastic approaches, which usually supply a good solution or until the global optimum; however, the computational time they require can exceeds that of deterministic methods (Wood (1996), Suram et al. (2005)). Other techniques based on artificial intelligence field, such as genetic algorithms and artificial neural networks, have been used for the solution of inverse problems (Ayhan et al. (2004), Sablani et al. (2005); Mariani and Coelho (2009a, 2009b), Silva et al. (2009)).

This paper presents a procedure to estimate apparent thermal diffusivity as a function of the moisture content for a range of numerical/experimental temperatures, using Differential Evolution and Particle Swarm Optimization for obtain parameters of piecewise function through of inverse method. The problem considered here is relevant in food processing operations, such as the analysis of transient heat transfer during the drying, cooling or freezing of products in continuous systems, which requires knowledge of the thermal properties of foods.

Thus the main objective of this work is to analyze and validate two stochastic optimization methods, Differential Evolution and Particle Swarm Optimization, applied in problem inverse for determination of apparent thermal diffusivity in the range between 20°C to 70°C for drying temperature. The second objective is to study heat and mass transfer aspects during drying process and use transient temperatures to estimate the apparent thermal diffusivity as a function of the moisture content.

The remainder of this paper is organized as follows: section 2 presents the heat mass transfer equations, while section 3 explain the fundamentals of inverse problem, optimization methods, and thermophysical properties. Subsequently, section 4 provides the analysis of results. Lastly, conclusion is given in the section 5.

2 Heat and Mass Transfer Equations

The method used to estimate apparent thermal diffusivity was based on the conduction heat transfer equation. To simplify the problem the following hypotheses were considered:

(i) The body is represented in the geometric form of an infinite cylinder of length L (m) and radius R (m) defined between [0; R], where $R \ll L$; thus, the longitudinal heat and moisture transfer were neglected and the axial symmetry was considered.

(ii) The thermal diffusivity was considered as a function of moisture content during drying.

(iii) The body is considered homogeneous.

By according to Figure 1, one of the boundaries is in contact with the surrounding air thus resulting in a convective boundary condition for both temperature and moisture content. The conservation equations proposed with associated initial and boundary conditions for the modeling of such physical problem involving the energy equation, based on Fourier's law and mass transfer equation described by Fick's unidirectional diffusion equation (Crank, 1975; Smith, 1985) are as follows:

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha\frac{\partial T}{\partial r}\right), \tag{1}$$

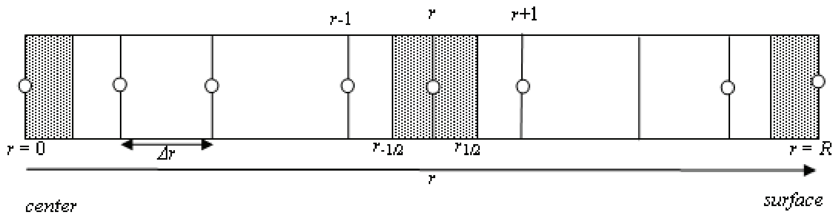

Figure 1: Computational domain.

$$\frac{\partial X}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{ef}\frac{\partial X}{\partial r}\right), \tag{2}$$

where α (m^2/s) is the thermal diffusivity, T (°C) is the internal temperature, D_{ef} (m^2/s) is the effective mass diffusivity, X (kg_w/kg_{dm}) is the moisture content (dry basis), r (m) is the transfer direction and t (s) is the time.

As initial condition, it was considered that initial temperature and moisture of the food are uniform, Eqs. (3) and (4). Symmetry conditions were considered at the banana geometric center, Eqs. (5) and (7). Convective and evaporative effects due to moisture and heat transfer at surface, Eqs. (6) and (8), were considered.

Initial conditions:

$$T(r,0) = T^0, \quad \forall r, \tag{3}$$

$$X(r,0) = X^0, \quad \forall r, \tag{4}$$

Boundary conditions:

$$\left.\frac{\partial T}{\partial r}\right)_{r=0} = 0, \tag{5}$$

$$-k\left.\frac{\partial T}{\partial r}\right)_{r=R} = h(T_R - T_e) + \rho_s \Delta r \frac{\partial \overline{X}}{\partial t}\left[h_{fg} + c_v(T_R - T_e)\right], \tag{6}$$

$$\left.\frac{\partial X}{\partial r}\right)_{r=0} = 0, \tag{7}$$

$$-D_{ef}\left.\frac{\partial X}{\partial r}\right)_{r=R} = h_m(X_R - X_e), \tag{8}$$

where k(W/m°C) is the thermal conductivity of the fruit, h(W/m^{2o}C) is the heat transfer convective coefficient, Δr is the spatial mesh step, ρ_s = 1970 (kg/m^3) is the dry solid density, h_{fg} (J/kg) represents the latent heat of vaporization of water, c_v (J/kg.K) is the specific heat of vapor of water, both obtained by air dry conditions $\overline{X} = \frac{1}{R}\int_0^R X(r,t)dr$ (kg_w/kg_{dm}) is the average moisture content in the section and h_m (m/s) is the mass transfer convective coefficient. Due to the characteristics of the mathematical problem (one-dimension and homogeneous material), the simpler finite difference technique (Crank, 1975) can be used rather than the finite element method or finite volume method for the solution of these partial differential equations. In this work, an explicit scheme was selected. Using this numerical scheme, the Eq. (1) can be described and approximated in the following terms,

$$\left(\frac{T_r^{t+\Delta t} - T_r^t}{\Delta t}\right) = \frac{\alpha}{2r\Delta r^2}\begin{bmatrix} r_{1/2}\left(T_{r+1}^t - T_r^t + T_{r+1}^{t+\Delta t} - T_r^{t+\Delta t}\right) - \\ r_{-1/2}\left(T_r^t - T_{r-1}^t + T_r^{t+\Delta t} - T_{r-1}^{t+\Delta t}\right) \end{bmatrix} \tag{9}$$

The Eq. (1) at r=0 can be replaced by $\frac{\partial T}{\partial t} = 2\frac{\partial}{\partial r}\left(\alpha\frac{\partial T}{\partial r}\right)$, thus at the food center (Figure 1) symmetric condition was considered where $T_{r+1} = T_{r-1}$, we can write this equation in the discretized form as follows:

$$\left(\frac{T_r^{t+\Delta t} - T_r^t}{\Delta t}\right) = \frac{2\alpha}{\Delta r^2}\left[T_{r+1}^t - T_r^t + T_{r+1}^{t+\Delta t} - T_r^{t+\Delta t}\right] \tag{10}$$

To boundary condition at the surface, since Eq. (6), the solid temperature can be calculated as follows,

$$T_R^{t+\Delta t} = \frac{\left[T_{R-1}^{t+\Delta t} + \frac{h\Delta r}{k}T_e - \frac{\rho_s \Delta r^2}{k}\left(\frac{\overline{X}^{t+\Delta t} - \overline{X}^t}{\Delta t}\right)\left(h_{fg} - c_v T_e\right)\right]}{\left[1 + \frac{h\Delta r}{k} + \frac{\rho_s \Delta r^2 c_v}{k}\left(\frac{\overline{X}^{t+\Delta t} - \overline{X}^t}{\Delta t}\right)\right]} \tag{11}$$

Substituting Eq. (11) into Eq. (9) we obtain the temperature $T_{R-1}^{t+\Delta t}$. The discretization of the Eq. (2) is omitted here due to its analogy with Eq. (1).

3 Inverse Problem

Knowing the food's geometry and physical properties, as well as the boundary and initial conditions, enables one to solve Eqs. (1) to (8), thus determining the transient temperature and mass distribution in the food. This type of problem is called a direct problem. If any of these magnitudes or a combination of them is unknown, but experimental data are available on the temperature measured inside and/or on the external surface of the food, we have an inverse problem that allows one to determine the unknown magnitudes, provided those data contain sufficient information.

The interest of the present work is to estimate the apparent thermal diffusivity using experimental data of the temperature obtained experimentally at the center of the banana during a time interval. In this work is desired to minimize the difference between experimental and predicted temperatures. Mathematically it is desired to minimize the objective function,

$$f = \sqrt{\frac{\sum_{j=1}^{n}(\tau_0^j - T_0^j(\alpha))^2}{n}}, \tag{12}$$

where T_0^j (°C) is the temperature of the banana at central node, r = 0, calculated numerically by the explicit finite difference method, j is the time indicator, τ_0^j (°C) is the experimental temperature of the banana at central thermocouple, $r = 0$, and n is the number of samples.

In most of the techniques developed to solve inverse problems, the numerical model must be able to solve the direct problem with values arbitrated to the magnitudes to be determined. Since the procedures for the solution are usually iterative, the direct problem must be solved several times. Thus, it is desirable to have a precise method for the solution of the direct problem that requires a relatively short computational time. The Differential Evolution and Particle Swarm Optimization approaches were used as the optimization technique and are described as follows.

3.1 Differential Evolution (DE)

Evolutionary algorithms are computer-based problem-solving systems of evolutionary computation area based on principles of evolution theory. The interest in evolutionary algorithms is increasing quickly, due to robust and powerful adaptive search mechanisms these algorithms. Evolutionary algorithms have been used in many problems, dealing with multidimensional and multimodal search. There are a variety of evolutionary models that have been proposed and studied, such as genetic algorithms, evolution strategy, evolutionary programming, genetic programming, and recently differential evolution that are referred as evolutionary algorithms. They share a common conceptual base of simulating the evolution of individual structures via selection and reproduction procedure. The basic idea is to maintain a population of candidate solutions that evolve under selective pressure that favors better solutions (Goldberg (1989), Bäck et al. (1997), Coelho and Mariani (2007)).

Differential Evolution (DE) is a population-based stochastic function minimized (or maximized) relating to evolutionary computation, whose simple yet powerful and straightforward features make it very attractive for numerical optimization. DE combines simple arithmetical operators with the classical operators of recombination, mutation and selection to evolve from a randomly generated starting population to a final solution.

DE differs from conventional genetic algorithms in its use of perturbing vectors, which are the difference between two randomly chosen parameter vectors. The DE algorithm was first introduced by Storn and Price (1995), and was successfully applied in the optimization of some well-known non-linear, non-differentiable and non-convex functions by Storn and Price (1997).

The different variants of DE are classified using the following notation: DE/ϕ/β/δ, where ϕ indicates the method for selecting the parent chromosome that will form the base of the mutated vector, β indicates the number of difference vectors used to perturb the base chromosome, and δ indicates the recombination mechanism used to create the offspring population. The bin acronym indicates that the recombination is controlled by various independent binomial experiments.

The fundamental idea behind DE is a scheme whereby it generates the trial parameter vectors. If the cost of the trial vector is better than that of the target, then the target vector is replaced by the trial vector in the next generation. The variant implemented in Matlab (MathWorks) was the DE/rand/1/bin, which involved the following steps and procedures:

Step 1: Parameter setup

The user chooses the parameters of population size, the boundary constraints of optimization variables, the mutation factor (*MF*), the crossover rate (*CR*), and the stopping criterion of maximum number of iterations (generations), G_{max}.

Step 2: Initialization of an individual population

Set generation $k = 0$. Initialize one population of $i = 1, .., M$ individuals (real-valued n-dimensional solution vectors) with random values generated according to a uniform probability distribution in the n dimensional problem space. These initial individual values are chosen randomly from within user-defined bounds (boundary constraints).

Step 3: Evaluation of the individual population

Evaluate the fitness value related to a objective function of each individual in the population.

Step 4: Mutation operation (or differential operation)

Mutation is an operation that adds one vector differential in the population according to the following Eq. (13),

$$z_i(k+1) = x_{i,r_1}(k) + MF \cdot [x_{i,r_2}(k) - x_{i,r_3}(k)] \tag{13}$$

where i=1,2,...,M is the individual's index of population; k is the generation (iteration); $x_i(k) = [x_{i_1}(k), x_{i_2}(k), ..., x_{i_n}(k)]^T$ stands for the position of the i-th individual of population of N real-valued n-dimensional vectors; $z_i(k) = [z_{i_1}(k), z_{i_2}(k), ..., z_{i_n}(k)]^T$ stands for the position of the i-th individual of one mutant vector; r_1, r_2 and r_3 are mutually different integers and also different from the running index, i, randomly selected with uniform distribution of the set $\{1, 2, \cdots, i-1, i+1, \cdots, N\}$; MF> 0 is a real parameter called mutation factor, which controls the amplification of the difference between two individuals so as avoid stagnation and is usually taken from the range [0.1, 1].

Step 5: Recombination operation

Following the mutation operation, recombination is applied to the population. Recombination is employed to generate a trial vector by replacing certain parameters of the target vector with the corresponding parameters of a randomly generated donor vector. For each vector, z_i(k+1), an index $rnbr(i) \in \{1, 2, \cdots, n\}$ is randomly chosen using uniform distribution, and a trial vector, $u_i(k+1) = [u_{i_1}(k+1), u_{i_2}(k+1), ..., u_{i_n}(k+1)]^T$, is generated with

$$u_{i_j}(k+1) = \begin{cases} z_{i_j}(k+1), & \text{if } randb(j) \leq CR \ or j = rnbr(i), \\ x_{i_j}(k), & \text{if } randb(j) > CR \ \ or \ \ j \neq rnbr(i). \end{cases} \tag{14}$$

In the above equations, randb(j) is the j-th evaluation of a uniform random number generation with [0, 1] and CR is a crossover or recombination rate in the range [0, 1]. The performance of a DE algorithm usually depends on three variables: the population size N, the mutation factor MF, and the recombination rate CR.

Step 6: Selection operation

Selection is the procedure of producing better offspring. To decide whether or not the vector $u_i(k+1)$ should be a member of the population comprising the next generation, it is compared with the corresponding vector $x_i(k)$. Thus, if f denotes the objective function under minimization, then

$$x_i(k+1) = \begin{cases} u_i(k+1), & \text{if } f(u(k+1)) < f(x_i(k)), \\ x_i(k), & otherwise. \end{cases} \tag{15}$$

In this case, the cost of each trial vector $u_i(k+1)$ is compared with that of its parent target vector $x_i(k)$. If the cost function (objective function), f, of the target vector $x_i(k)$ is lower than that of the trial vector, the target is allowed to advance to the next generation. Otherwise, the target vector is replaced by trial vector in the next generation.

Step 7: Verification of stop criterion

Set the generation number for $k = k + 1$. Proceed to Step 3 until a stopping criterion is met, usually G_{max}. The stopping criterion depends on the type of problem.

Each optimization approach was implemented in environment computational Matlab (MathWorks). To illustrate the effectiveness of the optimization procedure several simulations were performed. The program was run on a 3.8 GHz Pentium IV processor with 2 GB of RAM. In the tests, 30 independent runs were made for the optimization method involving 30 different initial trial solutions. In optimization tests, the setup of DE used was the following: $MF = 0.3$, $CR = 0.8$, the population size N was 10 and the stopping criterion G_{max} was 200 generations for the DE.

3.2 Particle Swarm Optimization (PSO)

The field of swarm intelligence is an emerging research area that presents features of self-organization and cooperation principles among group members bio-inspired on social insect societies (Dorigo and Stützle (2004), Kennedy et al. (2001), Bonabeau et al. (1999)). Swarm intelligence is inspired by nature, based on the fact that the live animals of a group contribute with their individual experiences to the group, rendering it stronger to face other groups.

The particle swarm optimization (PSO) originally developed by Kennedy and Eberhart (1995) is a population-based swarm algorithm. Similarly to genetic algorithms (Goldberg, 1989), PSO is an optimization tool based on a population, where each member is seen as a particle, and each particle is one potential solution to the problem under analysis. Each particle in PSO has a randomized velocity associated to it, which moves through the space of the problem. However, unlike genetic algorithms, PSO does not have operators, such as crossover and mutation. PSO does not implement the survival of the fittest individuals; rather, it implements the simulation of social behavior (Coelho and Mariani, 2008). The procedure for global version of PSO is given by the following steps:

Step 1: Initialization of swarm positions and velocities:

Initialize a population (array) of particles with random positions and velocities in the n dimensional problem space using uniform probability distribution function.

Step 2: Evaluation of particle's fitness:

Evaluate each particle's fitness value.

Step 3: Comparison to pbest (personal best):

Compare each particle's fitness with the particle's pbest. If the current value is better than pbest, then set the pbest value to be equal to the current value, and the pbest location to be equal to the current location in n-dimensional space.

Step 4: Comparison to gbest (global best):

Compare the fitness with the population's overall previous best. If the current value is better than gbest, then reset gbest to the current particle's array index and value.

Step 5: Updating of each particle's velocity and position:

Change the velocity, v_i, and position of the particle, x_i, according to:

$$v_i(t+1) = w \cdot v_i(t) + c_1 \cdot ud \cdot [p_i(t) - x_i(t)] + +c_2 \cdot Ud \cdot [p_g(t) - x_i(t)] \quad (16)$$

$$x_i(t+1) = x_i(t) + \Delta t \cdot v_i(t+1) \quad (17)$$

where w is the inertia weight; i=1,2,...,N indicates the number of particles of population (swarm); t=1,2,..., t_{max}, indicates the iterations, $v_i = [v_{i1}, v_{i2},...,v_{in}]^T$ stands for the velocity of the i-th particle,$x_i = [x_{i1}, x_{i2},...,x_{in}]^T$ stands for the position of the i-th particle of population, and $p_i = [p_{i1}, p_{i2},...,p_{in}]^T$ represents the best previous position of the i-th particle. Positive constants c_1 and c_2 are the cognitive and social components, respectively, which are the acceleration constants responsible for varying the particle velocity towards *pbest* and *gbest*, respectively. Index g represents the index of the best particle among all the particles in the swarm. Variables *ud* and *Ud* are two random functions in the range [0, 1]. Equation (17) represents the position update, according to its previous position and velocity, for$\Delta t = 1$.

Step 6: Repeating the evolutionary cycle:

Return to Step 2 until a stop criterion is met, usually a sufficiently good fitness or a maximum number of iterations (generations).

3.3 Thermophysical Properties

Experimental results for temperature used in this study were obtained from Pérez (1998), whose work presents results for six banana drying experiments with different conditions of velocity, temperature and relative humidity to air, which are presented in Table 1, where X_0 is the initial moisture content (kg_w/kg_{dm}) and X_e is the equilibrium moisture content (kg_w/kg_{dm}).

The values for heat transfer convective coefficient were obtained based on the Nusselt number,$h = kNu/d$, where k (W/mK) is the air thermal conductivity, d (m) is the diameter of the banana, Nu is the Nusselt number given by $Nu = 0.97 + 0.68Re^{0.52}Pr^{1/3}$, Pr is the Prandtl number, Re is the Reynolds number calculated by $Re = \rho vd/\mu$, ρ (kg/m^3) is the air density and μ (Pa.s) is the air viscosity. The numerical simulations were performed for values of the heat transfer convective coefficient, h, calculated from Eq. (18), in the range between 15 and 35 W/m^{2o}C, while the values of air velocities, v, are in the range between 0.33 and 0.39 m/s.

The shrinkage phenomenon was included in this work through of an empirical equation obtained from experimental results:

$$R = \left[0.4721 + 0.1819X_e + 0.1819(\overline{X} - X_e)\right] R_0, \tag{18}$$

Numerically, the shrinkage was treated like an elastic grid. This means that the number of nodes in the radius was maintained constant and the radial subinterval size was changed at each time step. The shrinkage is strong and fast at surface since X decrease quite fast main at the beginning of the drying.

Table 1: Air drying conditions and parameters used in the experimental tests.

Test	T_e(°C)	R (m)	X_0 (kg_w/kg_{dm})	X_e(kg_w/kg_{dm})	t(h)
1	29.9	0.01613	3.43	0.1428	121.9
2	39.9	0.01569	3.17	0.0664	72.0
3	49.9	0.01522	3.21	0.0579	40.8
4	60.2	0.01530	2.96	0.0426	35.3
5	60.5	0.01506	3.04	0.0211	27.8
6	68.4	0.01545	2.95	0.0121	27.6

Note that several authors have derived equations to predict thermal properties. Semi-theoretical equations (Krokida et al. (2001), Maroulis et al. (2002)) are simple to use however these equations are not always in agreement with experimental data. The functional forms of thermal properties are generally unknown, especially in the case of foods with multiple compositions. A preliminary choice of these functions could be an obstacle to a correct approximation of these thermal properties dependent of the temperature and/or moisture, even if the parameters of these functions are adjustable. The usual solution consists of representing these functions by empirical polynomials, some authors have proposed replacing polynomials by piecewise linear functions of temperature, and to adjust the parameters by optimization approaches.

In this study it was proposed the use of a non-linear function dependent of dimensionless average moisture content in section. The parameters were adjusted by inverse method using a DE and PSO approaches, thus the number of parameters for adjust is three, using the following equation

$$\alpha(\overline{X}^*) = \frac{A_1}{A_2^{\overline{X}^*} + A_3}, \tag{19}$$

where apparent thermal diffusivity is dependent of dimensionless average moisture content in the section, $\overline{X}^* = \frac{\bar{X} - X_e}{X_o - X_e}$.

4 Results and Discussion

The mathematical model described in Eqs. (1) to (8) considers some strict assumptions (homogeneous material and infinite cylinder). To characterize the apparent thermal diffusivity variable as a function of the moisture content, several preliminary analyses were made. The best fitness of the least square sense, between experimental and computational temperature is shown in Table 2. Deviations between experimental and simulated temperatures were calculated using the multiple correlation coefficient (Pearson coefficient), in successive trials as,

$$R^2 = 1 - \frac{\sum\left(\tau_0^j - T_0^j(\alpha)\right)^2}{\sum\left(\tau_0^j - \overline{\tau_0^j}\right)^2} \tag{20}$$

where $\overline{\tau_0^j}$ (°C) is the mean experimental temperature of the banana at thermocouple central, r = 0, and j is the time indicator. The R^2 value of 0.9 to 1.0 is considered sufficient for that the apparent thermal diffusivity obtained in this work to be well adjusted with the experimental data. In Table 2 are shown the values of the constants presents in the equation (19) obtained in this work. The values for goal function are shown in the last column in same table.

Table 2: Parameters of the Eq. (19) obtained using DE.

Cases	$A_1.10^{11}$	A_2	A_3	R^2	f
1	6.3197	0.3027	- 0.3000	0.9973	0.09
2	9.7950	0.3674	- 0.3658	0.9976	0.16
3	12.0060	0.4835	-0.4828	0.9994	0.04
4	11.5770	0.5000	- 0.4994	0.9982	0.21
5	17.6650	0.4689	-0.4673	0.9996	0.05
6	19.6654	0.4155	- 0.4141	0.9999	0.07

In Table 2 ones observe, through of R^2 values that the results predicted using DE optimization method have a good agreement with experimental values, showing that the apparent thermal diffusivity obtained from the inverse method was fitted by function presented in Eq. (19). In Table 3 the values for the same cases are presented using PSO method.

Figure 2 shows predicted and experimental temperatures at the thermal centre (r = 0) predicted by Eq. (1) using Eq. (19) using the parameters of the Table 2 for all cases. The predicted temperatures are in excellent agreement with experimental data obtained in Pérez (1998). Predicted temperatures were calculated with

Table 3: Parameters of the Eq. (19) obtained using PSO.

Cases	$A_1.10^{11}$	A_2	A_3	R^2	f
1	5.4280	0.4857	- 0.4839	0.9989	0.06
2	9.5420	0.4077	- 0.4062	0.9983	0.13
3	13.391	0.3441	-0.3432	0.9997	0.09
4	15.967	0.3475	- 0.3450	0.9910	0.48
5	19.988	0.3022	-0.3000	0.9999	0.07
6	20.520	0.4943	- 0.4920	0.9932	0.67

parameters estimated under the inverse method using the Differential Evolution. Statistical analysis through of the values of multiple correlation coefficients shows no significant differences between reported and estimated curves (see Table 2). In this figure the shrinkage and convective and evaporative effects at banana surface are included in the mathematical model, so, this is a complete model due the incorporation of physical phenomena in the banana's drying. It is important to observe that in practice, bananas shrink by about 43 $\pm$ 47% their original diameter during drying. This fact reveals the importance of including this phenomenon in the theoretical model. The inclusion of shrinkage and convective and evaporative effects lends more credibility to the apparent thermal diffusivity obtained and presented in Table 2. The minimum and maximum values for apparent thermal diffusivity obtained in this work using Eq. (19), for example, for fourth case were 2.49×10^{-10} (m^2/s) and 1.88×10^{-7} (m^2/s), respectively.

Figure 3 presents the comparative box plots of the data obtained from the objective function using two optimization methods, DE and PSO, after 30 runs for the case 3. From Figure 3, we can see that the results using PSO presented more variability than the results based on DE method. The PSO method needs to improve its convergence performance and design setup. Furthermore, the box plot to the DE method indicates that the objective function distribution is reasonably symmetric around the central value, median, there is a smooth outlier in the upper end of the data.

Figure 4 provides the comparative box plots obtained for the PSO and DE based on 30 independent runs for the case 5. Note in the Figure 4 again that the data obtained with the PSO method has greater dispersion than the data obtained with the DE method. The diagram of box representing the objective function of the data obtained by the PSO method indicates that the distribution is not symmetrical, the median value is high and there are two outliers in the lower end of the data. Already, the box plot to represent the performance of the DE method indicates a less dispersed distribution, however also presents a smooth outlier in the upper end

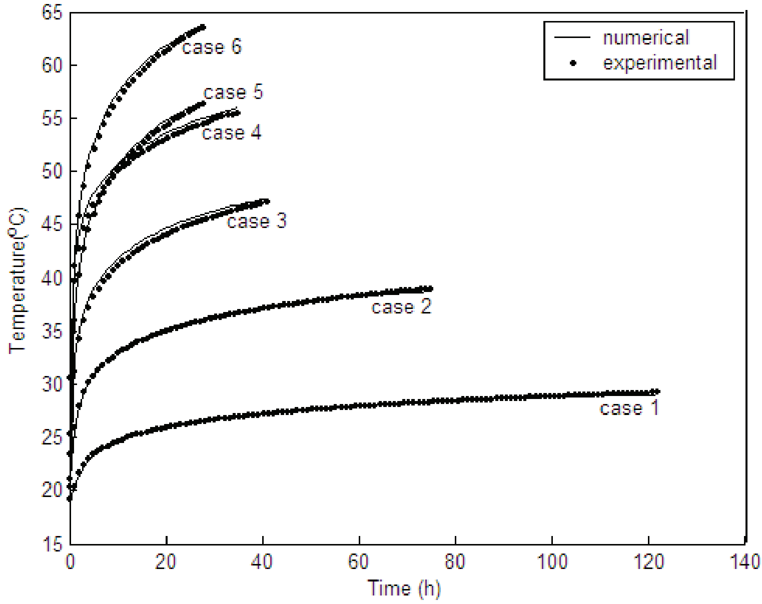

Figure 2: Experimental validation for the inverse method using the parameters of the Table 2 (central temperature, in r = 0).

of the data. So for this study the DE method had behavior less dispersed that the PSO method. Therefore, from the above observations, it is clear that validated DE method has a better performance that the PSO method.

5 Conclusion

In this study, Differential Evolution and Particle Swarm Optimization methods were successfully applied to the determination of apparent thermal diffusivity as a function of the dimensionless average moisture content in the section radial of the banana during the drying process. A statistical analysis shows no significant differences between the predicted and experimental profiles of temperature at the thermal centre of the banana, and DE method has a better performance that the PSO method. The results obtained in this work validate the proposed method as a tool to the determination of apparent thermal diffusivity in the drying temperature range. The proposed procedure can be extended to the determination of other thermal properties in different processes like thermal conductivity, and specific heat in drying, wetting, cooling, heating and/or freezing. The determination

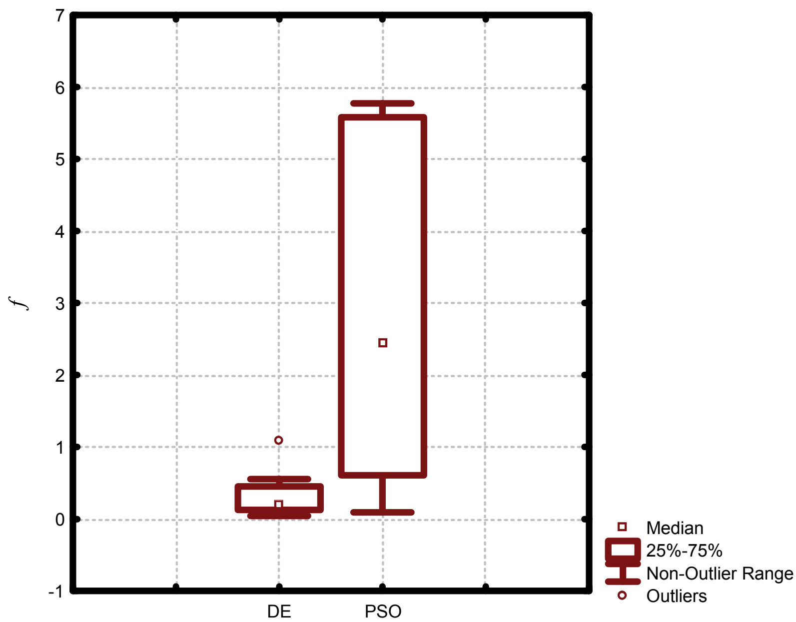

Figure 3: Statistical analysis using DE and PSO for case 3.

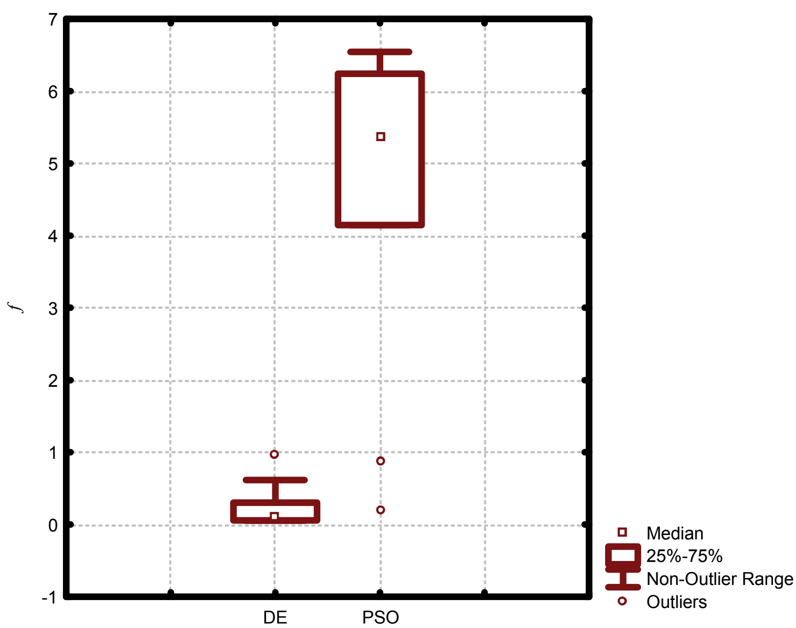

Figure 4: Statistical analysis using DE and PSO for case 5.

of thermal properties from an inverse method is an attractive technique both from the experimental and methodological point of view, because of its accuracy and short time for parameters estimation. The higher value obtained in this work to apparent thermal diffusivity was approximately 1.88×10^{-7} (m^2/s) while the lower value was 9.47×10^{-11} (m^2/s) when the moisture content changes from 2.95 to 3.43 (kg_w/kg_{dm}) and the air temperature changes from 29.9 to 68.4 (°C).

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