Analyzing Production-Induced Subsidence using Coupled Displacement Discontinuity and Finite Element Methods

Shunde Yin[1], Leo Rothenburg[1] and Maurice B. Dusseault[1]

Abstract: Subsidence problem is of great importance in petroleum engineering and environmental engineering. In this paper, we firstly apply a hybrid Displacement Discontinuity-FEM modeling to this classic problem: the evaluation of subsidence over a compacting oil reservoir. We use displacement discontinuity method to account for the reservoir surrounding area, and finite element methods in the fully coupled simulation of the reservoir itself. This approach greatly reduces the number of degrees of freedom compared to an analyzing fully coupled problem using only a finite element or finite difference discretization.

Keyword: Petroleum reservoir modeling, ground surface subsidence, geomechanics, displacement discontinuity boundary element method, finite element method

1 Introduction

In petroleum engineering and environmental engineering, evaluation of reservoir compaction and induced land subsidence is an active research area related to both offshore problems (Ekofisk, Valhall, Eldfisk in the North Sea) and on land (Groeningen, Niigata, Ravenna, etc.). The fluid-solid coupling during this process makes the numerical simulation much more complicated than, for example, fully drained or fully undrained processes.

1.1 Subsidence Issues in Petroleum Engineering

In petroleum engineering, large-scale reservoir compaction due to oil and gas withdrawal (i.e. $\Delta p \rightarrow \Delta\sigma\prime \rightarrow \Delta V \rightarrow \Delta z$) can lead to surface damage (Wilmington oil field, California; Lago de Maracaibo, Venezuela; Niigata, Japan; Ravenna, Italy), casing damage, and even well failure[Bruno (1992)]. The North Sea offshore oil field Ekofisk, developed in the early 1970's, experienced massive subsidence (4.3 m by 1988) so that all five platforms had to be raised in 1988-1990 at a cost of US$485,000,000, and fully redeveloped with two new platforms replacing the original five in the late 1990's at an additional cost in excess of US$3,000,000,000. Currently, reservoir compaction at Ekofisk appears to be ∼12 m, and sea floor subsidence has passed 10 m.

During massive subsidence, formation shear of many centimeters occurs at lithostratigraphic interfaces such as sandstone-shale or limestone-shale contact surfaces [Dusseault, Bruno and Barrera (2001)]. As an analogy, one may consider the shearing of the laminated surfaces in a sheet of plywood as it is sharply bent; delamination occurs because the shear strains are concentrated at the boundaries of materials of sharply contrasting stiffness (glue and wood). Sedimentary overburden is a laminated system with stiffness contrasts that cause shear deformation to be concentrated on a limited number of slip planes, rather than distributed uniformly throughout the volume. No matter how strong, a rigid cemented casing system penetrating such an interface at a large angle (>30°) cannot resist the shear forces, and will deform and rupture. In compacting reservoirs developed with vertical production wells, it is common to redrill wells several times to sustain production because of shearing-induced well losses.

Therefore, the numerical simulation of subsidence processes has attracted considerable interest over the decades, starting with the work of Geertsma (1966), and of course more recent work involving fully coupled three-dimensional

[1] University of Waterloo, Waterloo N2L 3G1, Canada

flow-deformation simulations incorporating non-linear behavior and temperature effects. Furthermore, the advent of high-precision deformation monitoring and inversion for reservoir processes [Dusseault and Rothenburg (2002)] is increasing the need for better models that are not only properly formulated, but rapid in execution, even for complex problems.

1.2 General Approaches for Computer Modeling in Petroleum Reservoir Engineering

To date, one may identify two types of schemes to implement flow-deformation coupling:

- A directly coupled FEM formulation in which displacements and pore pressure are assembled in into one system equation[e.g. see Lewis and Schrefler (1998)]; and,
- The staggered solution method, in which two separate systems (fluid flow and solid deformation) are computed separately and iteratively, with each providing coupling variables that the other needs for the solution[e.g.see Settari and Mourits(1998)].

For the former scheme, finite element methods are most commonly adopted and have proven robust; for the latter scheme, the fluid flow system is solved using finite difference methods based on four decades of petroleum industry simulation development, and the solid field solution generally employs finite element methods derived from mechanical engineering principles and modified for geomechanics applications.

In the conventional reservoir engineering treatment of this problem, only the transport problem in the reservoir is solved. This is analogous to assuming that the overburden has no stiffness and the total vertical stress on the upper surface of the reservoir is unaffected by pressure changes in the reservoir; i.e., it is assumed that $\Delta p = \Delta\sigma\prime_v$. This is clearly inadequate, as any change in pressure must be accompanied by a change in volume ($\Delta V/V = C\cdot\Delta\sigma\prime$, where C is compressibility); therefore, for any non-uniform pressure change in the reservoir, $\Delta p \neq \Delta\sigma\prime_v$, because of the stiffness of the overburden. An analytical solution presented by Rothenburg, Bratli and Dusseault (1994) for transient two-dimensional radial flow of a compressible fluid into a vertical penetrating well shows clearly that the stiffness of the strata, both reservoir and overburden, is an essential coupling element which must be taken into account. In the limit, for full coupling of flow and stress in the reservoir zone, the complete mechanical response of the overburden must be accounted for, even factors such as stratification and anisotropic properties.

In this paper, to achieve coupling in an efficient manner, we attempt to exploit the advantages of the displacement discontinuity boundary element method in solving the stress-strain problem in infinite and semi-infinite domains, combined with finite element methods for solving the flow-deformation system in the reservoir.

2 Half-Space Simulation and the Displacement Discontinuity Method

2.1 Surface Deformation Problems in a Half Space

Researchers [e.g. Gambolati and Freeze(1973), Schrefler, Lewis and Norris (1977), Sandhu(1983)] have successfully used the finite element method to analyze subsidence associated with fluid withdrawal from a underground petroleum reservoir or aquifer in the recent several decades. For the true half-space problem, most researchers have used analytical or semi-analytical techniques. Geertsma(1966) developed a semi-analytical solution for the prediction of subsidence by using the displacement solution for a thermoelastic nucleus-of-strain in a half space with a traction free surface [Mindlin and Cheng (1950)]. Segall(1992) presented a more systematic approach to calculate the displacements and stresses caused by fluid extraction in a axisymetric reservoir within the framework of linear poroelasticity . Rothenburg, Obah and El Baruni (1995) presented a solution for horizontal ground movements and the formation of earth fissures due to water table decline based on a nucleus-of-strain solution for a poroelastic half space.

Some researchers have tried numerical methods for this problem. For example, Gambolati, Sartoretto, Rinaldo and Ricceri(1987) proposed a linear boundary element model (BEM) for the uncoupled simulation of land subsidence due to gas, oil and hot water production over three-dimensional reservoirs. Their model allows for arbitrary reservoir geometry and for non-uniform pore pressure changes. Suzuki and Morita (2004) used a 3D BEM to analyze surface subsidence and lateral movement due to uniform pore pressure decline during oil and gas production.

Comparing the shape and size of a typical reservoir to the whole domain, we can consider the reservoir as a displacement discontinuity [Charlez (1997), Rothenburg, Bratli and Dusseault (1994)], and therefore use a different indirect boundary element method (IBEM), the displacement discontinuity method (DDM), to replace the boundary element method (BEM).

2.2 Displacement Discontinuity Boundary Element Method (DDM)

Boundary element method (BEM) is a numerical analysis technique for boundary value problems based on integral equation formulations, as opposed to the differential equation formulations which underpin the finite element (FEM) and finite difference (FDM) methods. One advantage of BEM is that it reduces the dimension of the problem by one with only the boundaries discretized. The other advantage is that BEM is able to satisfy far field boundary conditions implicitly, and this make it very powerful dealing with infinite and semi infinite media. There are two basic BEM approaches, the indirect BEM which use fictitious quantities as source densities and the direct BEM which is formulated in terms of physical quantities such as displacements and tractions.

Applications of BEM in geomechanics have been extensively in the areas as follows: mining, excavation, tunneling, consolidation, ground water flow, soil-fluid-structure interaction, and fracture propagations.

The displacement discontinuity method is an indirect boundary element method for solving problems in solid mechanics. In geomechanics, it is usually used for analyzing large scale mining layouts [Salamon (1963)] in infinite or semi-infinite media, and it is useful in cases involving displacements along faults or joints, in fracture mechanics, and for simulating mining in tabular ore bodies (which extend at most a few meters in one direction and hundreds or thousands of meters in the other two). An advantage of the displacement discontinuity method for problems in geomechanics, like any boundary method, is that boundary conditions at infinity are automatically satisfied. Hence, full domain discretization and stipulation of boundary conditions on non-infinite boundaries can be avoided. Inspired by the similarities between a tabular ore body and the typical tabular reservoir in an oil field, we may consider applying this highly efficient method to the area outside the reservoir.

2.3 DD Mathematical Equations

In mining problems, the displacement discontinuity has been defined as the relative displacement between the roof and floor of a small area of a seam-like deposit. Similarly, for the behavior of a producing petroleum reservoir, the displacement discontinuity components can be defined as the relative displacement components between the top and bottom of a small area of a tabular reservoir. Consider a displacement discontinuity as a plane crack with a normal in the x_3 direction; its two faces can be distinguished by specifying one in the positive side ($x_3 = 0^+$) and the other is in the negative side ($x_3 = 0^-$). In crossing from one side to the other, the displacements undergo a specified change in value $D_i = (D_1, D_2, D_3)$ given by

$$\begin{aligned} D_1(x_1,x_2,0) &= u_1\left(x_1,x_2,0^-\right) - u_1\left(x_1,x_2,0^+\right) \\ D_2(x_1,x_2,0) &= u_2\left(x_1,x_2,0^-\right) - u_2\left(x_1,x_2,0^+\right) \\ D_3(x_1,x_2,0) &= u_3\left(x_1,x_2,0^-\right) - u_3\left(x_1,x_2,0^+\right) \end{aligned} \tag{1}$$

The general form solution for a displacement discontinuity element can be expressed as below [Salamon (1963), Crouch and Starfield (1983)]:

$$
\begin{aligned}
u_1 =& [2(1-v)\phi_{1,2} - x_3\phi_{1,13}] - x_3\phi_{2,12} \\
& - [(1-2v)\phi_{3,1} + x_3\phi_{1,13}] \\
u_2 =& [2(1-v)\phi_{2,3} - x_3\phi_{2,22}] - x_3\phi_{1,12} \\
& - [(1-2v)\phi_{3,2} + x_3\phi_{3,23}] \\
u_3 =& [2(1-v)\phi_{3,3} - x_3\phi_{3,33}] \\
& + [(1-2v)\phi_{1,1} - x_3\phi_{1,13}] \\
& - [(1-2v)\phi_{2,2} - x_3\phi_{2,23}] \\
\sigma_{11} =& 2G\{[2\phi_{1,13} - x_3\phi_{1,111}] \\
& + [2v\phi_{2,23} - x_3\phi_{2,112}] \\
& + [\phi_{3,33} + (1-2v)\phi_{3,22} - x_3\phi_{3,113}]\} \\
\sigma_{22} =& 2G\{[2v\phi_{1,13} - x_3\phi_{1,122}] \\
& + [2\phi_{2,23} - x_3\phi_{2,222}] \\
& + [\phi_{3,33} + (1-2v)\phi_{3,11} - x_3\phi_{3,223}]\} \\
\sigma_{33} =& 2G\{-x_3\phi_{1,133} - x_3\phi_{2,233} \\
& + [\phi_{2,33} - x_3\phi_{2,333}]\} \\
\sigma_{12} =& 2G\{[(1-v)\phi_{1,23} - x_3\phi_{1,112}] \\
& + [(1-v)\phi_{2,13} - x_3\phi_{2,122}] \\
& - [(1-2v)\phi_{3,12} + x_3\phi_{3,123}]\} \\
\sigma_{23} =& 2G\{[-v\phi_{1,12} - x_3\phi_{1,123}] \\
& + [\phi_{2,23} + v\phi_{2,11} - x_3\phi_{2,223}] - x_3\phi_{3,123}\} \\
\sigma_{13} =& 2G\{[\phi_{1,33} + v\phi_{1,22} - x_3\phi_{1,113}] \\
& + [-v\phi_{2,12} - x_3\phi_{2,123}] - x_3\phi_{3,133}\}
\end{aligned} \tag{2}
$$

where $\phi_{i,j}, \phi_{i,jk}, \phi_{i,jkl}\,(j,k,l = 1,2,3)$, are the derivatives of the kernel function

$$
\phi_i(x_1,x_2,x_3) = \frac{1}{8\pi(1-v)} \cdot \iint_{\Re} D_i \left[(x_1-\xi)^2 + (x_2-\eta)^2 + x_3^2\right]^{-1/2} d\xi d\eta \tag{3}
$$

in which $\Re$ is the area of the element, D_i $(I = 1,2,3)$ are the displacement discontinuities, (x_1,x_2,x_3) is the coordinate system originated at the element, and $(\xi,\eta,0)$ are the coordinates of the loading point. For the constant strength element, the displacement discontinuities can be taken out of the integration formula. Last equation is in terms of the basic kernel function

$$
I(x_1,x_2,x_3) = \iint_{\Re} \left[(x_1-\xi)^2 + (x_2-\eta)^2 + x_3^2\right]^{-1/2} d\xi d\eta \tag{4}
$$

which depends on the geometry of the element. The kernel functions were derived for the rectangular element by Salamon (1963).

3 Compaction-Subsidence Problems and the Finite Element Method

3.1 Theory of Poroelasticity

The theory of poroelasticity is the basis for developing the numerical formulation that will be used to study reservoir compaction and induced surface subsidence. The term poroelasticity was first coined by Geertsma (1966), referring to Biot's theory [Biot (1941)] of three dimensional consolidation. The earliest attempt to account for the influence of pore fluid on the quasi-static deformation of soils is the one-dimensional consolidation model of Terzaghi (1923), shown by Biot (1941) to be a special case of his more general theory. However, because of the complexity of the coupled set of partial differential equations, most analytical solutions of Biot's model are limited to simple geometries and boundary conditions.

Numerical techniques are required for more complex situations. Sandhu and Wilson (1969) first applied the finite element method to poroelasticity, and over the years many refinements and extensions have been made [Gambolati and Freeze(1973), Zienkiewicz and Talor (1991)]. Cheng and Detournay (1988) presented a direct boundary element method for plane strain poroelasticity.

In the following sections, we present a poroelastic finite element formulation for the reservoir zone and show how we treat the surrounding strata using the displacement discontinuity method.

3.2 Governing Equations of Poroelasticity

Based on the Biot's theory of poroelasticity and Darcy's law[Biot (1941)], simply, with the compressible fluid flowing through the saturated porous medium considered, the governing equations for the problem of oil flow in deforming reservoir rock can be described as (the body force is ignored):

$$G\nabla^2 u + (G+\lambda)\nabla\, \mathrm{div}\mathbf{u} - \alpha\nabla p = 0$$

$$\alpha\, \mathrm{div}\mathbf{u}_t + \left(\frac{1-\phi}{K_m} + \frac{\phi}{K_f} - \frac{1}{(3K_m)^2} i^T D i\right) p_t - \frac{k}{\mu}\nabla^2 p = 0 \quad (5)$$

where G and λ are Lame constants. k is the permeability of porous media, μ is the viscosity of fluid, u and p denote the displacement of porous medium and the pore pressure respectively, the subscript t denotes time derivative, ϕ is the porosity of the porous medium(i.e. percentage of pore volume within rock that can contain fluids), K_f and K_m is the bulk modulus of the fluid and matrix, respectively. $I^T = [1,\ 1,\ 1,\ 0,\ 0,\ 0]$.

3.3 Finite Element Solutions

Galerkin finite element method is chosen here to approximate the governing equations [Zienkiewicz and Talor (1991)]. The final form of the FE solution to the poroelastic equations is as follows:

$$\begin{bmatrix} M & -C \\ 0 & H \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ C^T & S \end{bmatrix} \begin{Bmatrix} \mathbf{u}_t \\ p_t \end{Bmatrix} = \begin{Bmatrix} f^{\mathbf{u}} \\ f^p \end{Bmatrix} \quad (6)$$

where M, H, S and C are the elastic stiffness, the flow stiffness, the flow capacity and coupling matrices, respectively.

$\{{}^{\mathbf{u}}_{p}\}$ and $\{{}^{\mathbf{u}_t}_{p_t}\}$ are the vectors of unknown variables $\mathbf{u}$ and p and corresponding time derivatives. f^u and f^p are the vector for the nodal loads and flow sources.

The explicit expressions of the above matrices are as follows.

$$M = \int_V B^T D B dV$$

$$H = \frac{k}{\mu}\int_V (\nabla N_p)(\nabla N_p)^T dV$$

$$S = \int_V N_p \left[\frac{1-\phi}{K_m} + \frac{\phi}{K_f} - \frac{1}{(3K_m)^2} I^T D I\right] N_p^T dV$$

$$C = \int_V B^T I N_p dV \quad (7)$$

4 Poroelastic Half-Space Subsidence Using Coupled DD–FEM

4.1 Coupling of Displacement Discontinuity and Finite Element Methods

Combining the advantages from both BEM and FEM has led to the hybrid BEM/FEM method, and its applications exist in many engineering sciences fields[Lie, Yu and Zhao(2001), Forth and Staroselsky(2005)].

In this section, we try to combine a FEM method for the reservoir with a DD formulation for the surrounding strata, to address the compaction induced surface subsidence problem numerically in a half-space domain. As far as we know, this is the first time for this hybrid approach to be applied addressing the production-induced subsidence problem in petroleum engineering and environmental engineering.

We employ the 20 node brick finite element and rectangular displacement discontinuity element in the implementation. Correlation between the DD element and FE element is shown in Figure 1.

The exchange of the information between the reservoir model and the DD model is performed. The information that the FEM model provides is the deformation of the reservoir, which is then converted into displacement discontinuity provided to the DD model; the information that the DD model provides is the stress state of the reservoir, which is then converted into overburdens provided to the FEM model.

The process of coupling between the reservoir model and the DD model is repeated until the convergence is achieved.

4.2 Implementation

We use the iteration method to implement data exchange between the DD and FEM models; the procedure is as follows. (The flowchart is shown in Figure 2.)

1. Start with the FEM (reservoir) model to calculate the displacement and pressure under prescribed external loads and fluid discharge conditions within a specified time period.

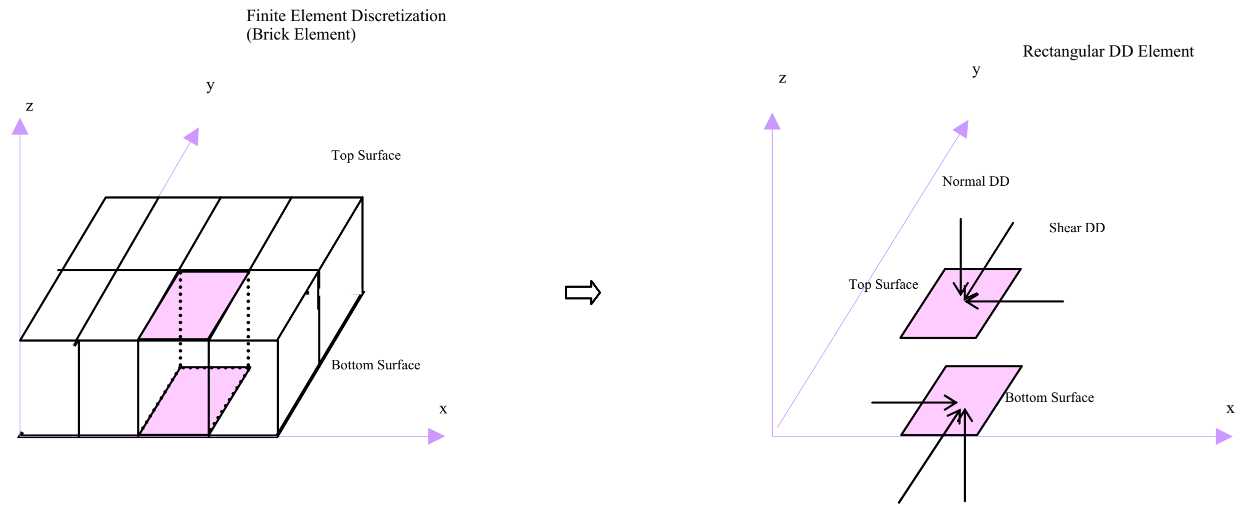

Figure 1: Sketch of relationship between Finite Element and Displacement Discontinuity Element

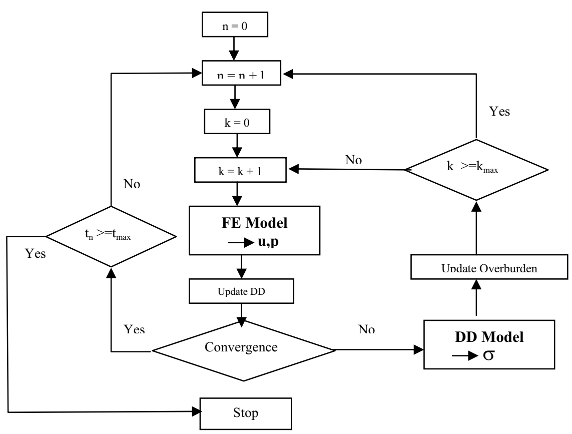

Figure 2: Flowchart of the iteration calculation between DD and FE Model

2. Convert displacements obtained from the FEM model into the displacement discontinuity which is needed to apply to the DD elements defining surrounding strata.

3. Execute the DD model, from which the local stresses can be computed.

4. Apply the induced stresses calculated from the DD model, along with the difference between the stresses in FEM and DDM, into the external loadings to be applied to the FEM model in the next iteration. The purpose of doing this is to make sure the stresses retain continuity.

Based on our experiences, to accelerate the convergence, we introduce a constant, χ, to multiply the stresses difference between the two model

$$\chi = \frac{E_r}{E_r + E_o} \tag{8}$$

where E_r represents the elastic modulus of the reservoir, E_o represents the elastic modulus of the surroundings.

5. The first iteration in the first time step ends, and this iteration is now repeated until the difference of the displacement discontinuity between successive iterations is less than the error tolerance. (There is also a criterion for maximum number of iterations.)

6. Now, the first time step is complete, and we return to step (1) above to undertake the second timestep, repeating the process until the desired time is reached. Intermediate stresses and displacements are stored for examination of time-dependent, diffusion-controlled factors.

4.3 Verification

Now, consider a 20 m × 20 m × 3 m reservoir at depth of 300 m(note that in reality it could be much deeper) with the following basic parameters (see Figure 3): E = 10 MPa, ν = 0.3, ϕ = 0.28,

$K_f = 1 \times 10^6$ kPa, $K_m = 1 \times 10^6$ kPa, k = 1 D, and μ = 1 cP.

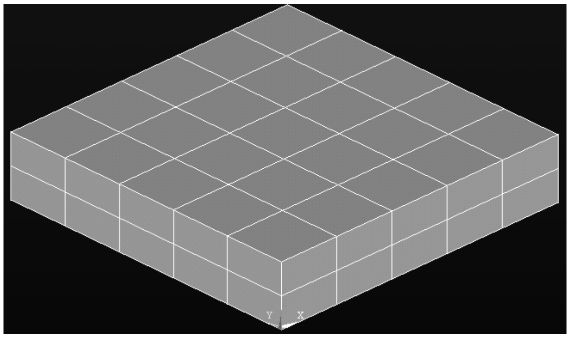

Figure 3: FEM Mesh of the reservoir model

The reservoir has a specified non-permeable boundary, and is supposed to be under a production rate of Q = 0.5 m^3/min with uniform pumping so that we can obtain a uniform pressure decline within the reservoir. This will allow us to compare results to Geertsma's analytical solution [Geertsma(1966)] for uniform drawdown. In the present problem, the time step is set as Δt = 120 minutes.

In the FEM mesh, the domain is discretized into 50 elements with 360 nodes; in the DD mesh, the domain is discretized into 25 DD elements. Convergence is shown in Figure 4.

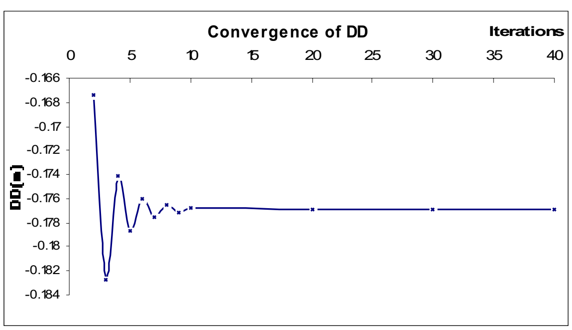

Figure 4: Convergence of the Displacement Discontinuity

Continuity of stress is demonstrated by the consistency of the stresses from both the FEM model and DD model (Figure 5).

The subsidence profiles at different time steps are shown in Figure 6, in which the series number 1, 2, 3, 4, and 5 represent 0, 2, 4, 6, and 8 hours respectively.

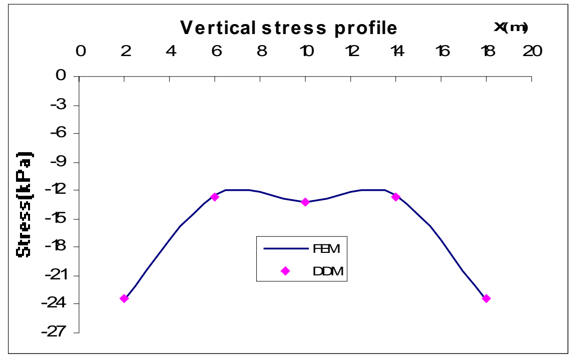

Figure 5: Continuity of Stresses between FEM and DDM models

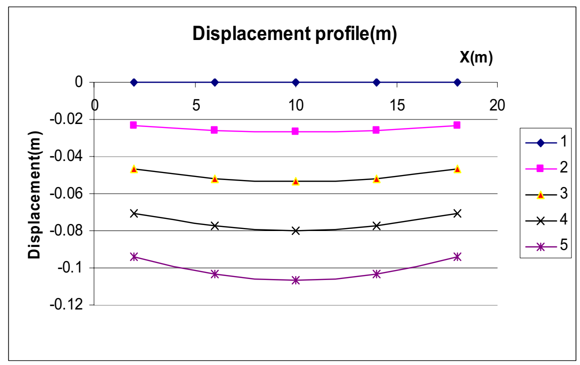

Figure 6: Subsidence profiles at different time

We will now compare the result of the DD-FEM model with Geertsma's analytical solution. He developed his analytical solution to predict subsidence caused by a uniform pressure decline in a fluid-saturated reservoir as follows: for r = 0 and a Δp which is constant throughout the reservoir, the vertical displacement can be expressed as:

$$u_z(0,z) = -\frac{c_m h}{2}\left[\frac{C(Z-1)}{[1+C^2(Z-1)^2]^{\frac{1}{2}}} - \frac{(3-4v)C(Z+1)}{[1+C^2(Z+1)^2]^{\frac{1}{2}}} + \frac{2CZ}{[1+C^2(Z+1)^2]^{3/2}} + (3-4v+\varepsilon)\right]\Delta p \quad (9)$$

where $Z = z/c$, $C = c/R$ and $\varepsilon = -1$ for $z > c$, and $\varepsilon = +1$ for $z < c$, respectively. Thus the elastic surface subsidence above the centre of a disc-

Table 1: Comparison between the Geertsma's Solution and FEM-DDM model

subsidence at the centre										
Time	E(kPa)	v	Thick(h)	~Radius(m)	Depth(m)	Drawdown(kPa)	Ratio(D/R)	Geertsma(m)	DDFEM(m)	Rel. Err.
2hrs	10000	0.3	4	11.283	300	-314.103	26.586808	-9.23E-05	-9.30E-05	0.68%
4hrs	10000	0.3	4	11.283	300	-635.232	26.586808	-1.87E-04	-1.86E-04	-0.35%
6hrs	10000	0.3	4	11.283	300	-957.137	26.586808	-2.81E-04	-2.79E-04	-0.77%
8hrs	10000	0.3	4	11.283	300	-1278.716	26.586808	-3.76E-04	-3.72E-04	-0.97%

shaped depleted reservoir amounts to

$$u_z(0,0) = -2(1-\nu)c_m h\Delta p \left(1 - \frac{C}{\sqrt{1+C^2}}\right) \tag{10}$$

Calculations based on the FEM-DD model and Geertsma's solution are compared in Figure 7; results from this hybrid method shows a high consistency with those of Geertsma.

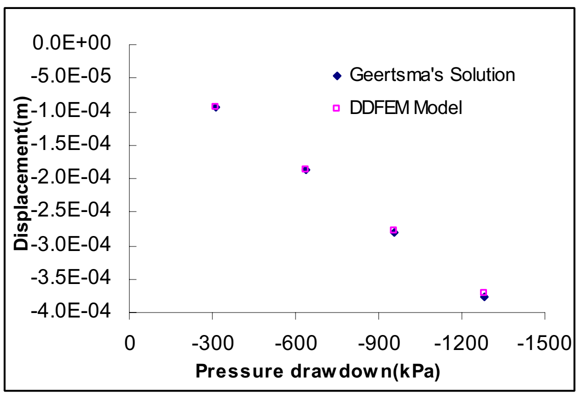

Figure 7: Comparisons between the Geertsma's Solution and FEM-DDM model

5 Conclusions

We presented a hybrid method that combined the advantages of both boundary element and finite element models to develop a coupled numerical simulation of a compacting reservoir within a semi-infinite half-space. Some of these advantages are:

1. It has advantages over analytical methods which are restricted to simple geometries, linear elastic rock behavior in the reservoir, uniform drawdown, etc.

2. It has advantages over the FEM method alone which must introduce proximal boundaries (not a true half-space) and leads to a much larger number of degrees of freedom for the discretization of the surrounding strata.

3. It has the advantage over the DD method alone, which cannot account for the flow-deformation coupling within the reservoir zone.

4. It has the advantage of higher accuracy with a reduced number of degrees of freedom through considering the reservoir compaction as one part of the problem, and its influence on the surrounding impermeable half-space domain as the second part of the problem. This seems to be a relatively natural way of addressing a large number of realistic problems.

Finally, through comparison with other methods and analytical solutions, we have shown that the DDFEM method leads to correct solutions.

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